Sovereign Bond Purchases and Rollover Crises[*]

Paul Fontanier[†]

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Abstract

This paper proposes a theory of large-scale government bond purchases by central banks in an environment with endogenous information acquisition. Information acquisition by private investors lowers risk premia by reducing uncertainty, but also makes prices more sensitive to new information. This can drive the sovereign into costly roll-over crises. Asset purchases by the central bank discourage private information acquisition, impairing price informativeness. This, however, points to a benefit of such large scale programs: by implementing purchases, the central bank can avoid the occurrence of roll-over crises in the event of bad news, generating large welfare gains. A key property of the model is that substantial purchases may be required, while small interventions have ambiguous welfare consequences. When the sovereign expects the central bank to carry such programs, it leads to excessive indebtedness, forcing the central bank to run an inflated balance sheet to avoid roll-over crises.

JEL Codes: E58, G14, H63

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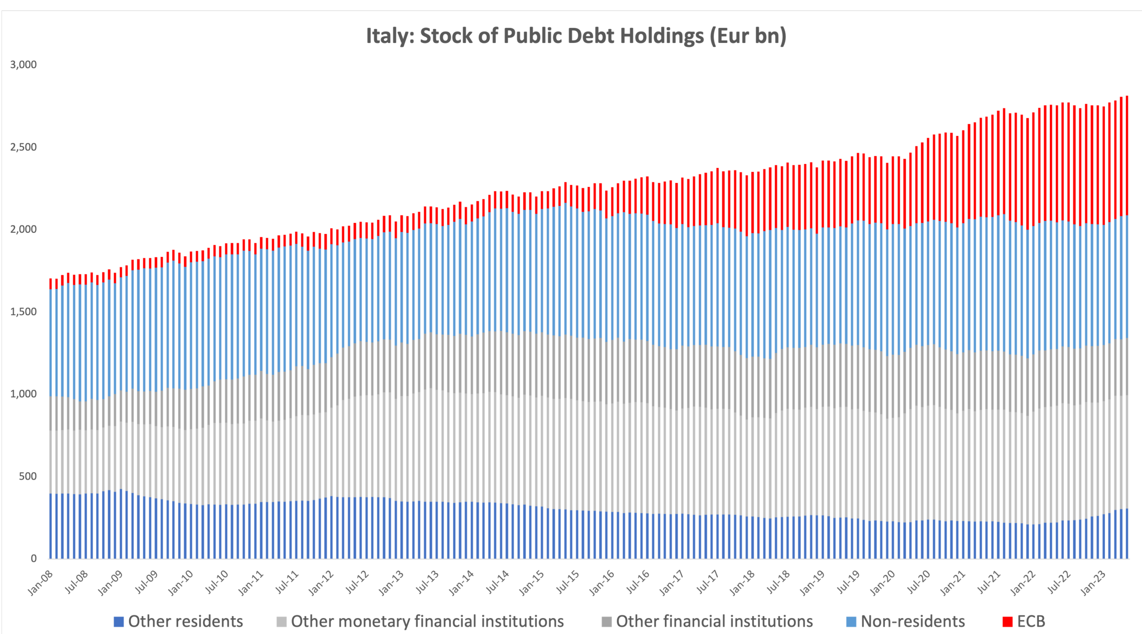
[†]Yale SOM. paul.fontanier@yale.edu & paulfontanier.github.io

1 Introduction

In January 2015, the ECB announced it would add the purchase of sovereign bonds to its existing private sector asset purchase programs. These purchases expanded further over time, so much that in April 2023 the ECB was holding more than a quarter of the total stock of Italian Government debt (Figure 1). The scale of this intervention, however, raises questions (Cochrane 2019 ; Bank for International Settlements 2019). While asset purchases can be a way to avoid negative self-fulfilling sovereign default, the benchmark view is that it is enough for a central bank to merely state its intention to make such purchases; it does not actually need to make them (Calvo 1988 ; Aguiar and Amador 2019).[1] Furthermore, asset purchases would seem to reduce the incentive of private investors to gather and aggregate information into prices, impairing the price discovery process in sovereign bond markets (Taylor 2014 ; Bond and Goldstein 2015 ; Lustig 2022). Why do central banks then sometimes conduct such large-scale purchases of sovereign debt?

Figure 1: Italian Sovereign Debt Holdings



[1]This is the common interpretation attached to the 2012 announcement by Mario Draghi, where he promised to do "whatever it takes" to preserve stability in the Eurozone. Without implementing any transaction, this sole announcement lowered the spreads of sovereign bonds issued by the distressed European countries.

This paper proposes a theory of sovereign debt issuance and default with information production, and provides a new rationale for bond purchases by the central bank. Information production by private agents has two natural effects: by reducing the uncertainty faced by investors, it lowers the risk premium on sovereign bonds. At the same time, it makes bond prices more sensitive to new information. By purchasing sovereign bonds directly, the central bank effectively transfers some risk away from private balance sheets, which reduces the incentives to invest in order to acquire information about future risks of default. As a consequence, asset purchases do indeed distort information production decisions, resulting in higher uncertainty and less sensitive bond prices.

In some cases, however, this paper shows that this is a feature of such programs rather than a bug. In the event where a large share of private investors acquires information, and this information is negative, bond prices can fall substantially. If the price drop exceeds some threshold level, this can in turn trigger self-fulfilling roll-over crises later on (Cole and Kehoe 2000) by making it too expensive for the sovereign to refinance itself. By purchasing enough sovereign bonds, the central bank can diminish information production such that the occurrence of bad news does not trigger roll-over crises anymore. Since these self-fulfilling events are extremely costly in welfare terms, it is optimal for the central bank to carry out these purchases, precisely to impair price discovery.

Section 2 presents the basic 3-period model used in the analysis. A government must issue (risky) bonds in period 1 to finance some expenditures. In periods 2 and 3, the government seeks to smooth consumption by rolling over some of its debt. When the government runs a new auction at time 2, it is vulnerable to self-fulfilling roll-over crises when the debt burden is higher than a certain level: if investors believe that the government is going to default on its newly issued debt, bond prices are zero, and it can then be optimal for the government to default also on its past debt (Cole and Kehoe 2000 ; Aguiar and Amador 2019). As a consequence, the country is subject to multiple equilibria when its debt issued in the previous period is priced below a threshold.[2] Taking this into account, mean-variance investors

[2]This model of roll-over crises seeks to capture the actions of the ECB with regards to Italy. Rising yields on Italian debt were a concern for the ECB as it worsened the financial situation of the country, leading to fear of roll-over crises down the road. The "anti-fragmentation tool" allows the

trade debt in the initial period. Investors can also invest ex-ante in order to acquire information about the (non-self-fulfilling) probability of default of the sovereign. By paying a fixed cost, investors receive a partially informative signal that allows them to trade with less uncertainty, and with an updated expectation of default.

When a larger share of investors is acquiring information, the risk premium on government bonds is lower, making it easier for the sovereign to finance its project on average. On the other hand, bond prices are now more sensitive to new information since a larger share of investors is trading on this information. When the signal received by informed investors is negative, and a large enough share of investors is informed, bond prices can fall below the threshold triggering roll-over crises and multiple equilibria in the second period. While optimal from the point of view of an isolated investor, information acquisition can thus be "excessive" from the point of view of the sovereign.[3]

The effect of asset purchases by the central bank in this environment is studied in Section 3. I start by showing that, below the threshold that triggers possible roll-over crises, asset purchases do crowd out private information production, consistent with recent critiques (Lustig 2022). This is intuitively because, taking into account the central bank's actions, investors realize that they will hold fewer bonds on their balance sheet, which makes it less worthwhile to pay a fixed cost. Because the central bank also transfers some risk away from private balance sheets (Caballero and Simsek 2021 ; Costain, Nuño and Thomas 2022), however, the effect on asset prices is likely to be positive on average (albeit ambiguous in general). The two effects compete against each other: less information acquisition increases the risk premium, but by having to hold less debt, investors also require a lower compensation. The second effect of these asset purchases is nevertheless unambigu-

ECB to implement secondary "market purchases of securities issued in jurisdictions experiencing a deterioration in financing conditions not warranted by country-specific fundamentals" (European Parliament 2022). In my model, these non-fundamental movements are interpret as yield increases caused by a fear of (self-fulfilling) roll-over crises.

[3]This feature of financial markets was noted by the 1988 Delors Committee, which set out the blueprint for the Economic and Monetary Union (EMU) (later structured in the Maastricht Treaty in 1992): "Market views about the creditworthiness of official borrowers tend to change abruptly and result in the closure of access to market financing. The constraints imposed by market forces might either be too slow and weak or too sudden and disruptive" (Delors Committee 1989). For a thorough analysis of the ECB market-based approach to government debt, see Van't Klooster (2023).

ous: by discouraging information acquisition, bond prices become less responsive to news.[4] This is crucial in the case where, without action by the central bank, information production is above the threshold and roll-over crises are thus possible in the event of negative news.

Section 4 applies these insights to understand the optimal asset purchase program the central bank should implement to maximize welfare. In all cases, small asset purchases have an ambiguous effect, as they have an ambiguous effect on asset prices. When information production in equilibrium is high enough that multiple equilibria are possible next period, large enough asset purchases can substantially improve the welfare of the country. This is because avoiding roll-over crises creates a welfare "jump." Because of this discontinuity, this result holds even when taking into account that it is costly for the central bank to expand its balance sheet. This size-dependence is the second key insight of this paper: the actions of the central bank must be large enough to shift the equilibrium information production from the right of the roll-over threshold to the left of it. This also contrasts my model to the well-known result that the central bank only need to be able to commit to intervene in order to avoid multiple equilibria, and thus never has to actually intervene (Aguiar and Amador 2019).

Section 5 presents a variety of extensions and robustness checks of the analysis. In particular, it shows that my results are not driven by the simplifying assumption that the government must issue bonds to finance a project. I show that the government can still willingly expose itself to roll-over crises, even while realizing that information production is going to be excessive (for usual consumption smoothing reasons). Asset purchases by the central bank ex-post will then have a similar impact on welfare. A crucial difference is that the sovereign can then anticipate that the central bank will step in, which also distorts debt issuance incentives. I show that this can result in much larger debt issuance levels, even if the government would normally issue debt at some lower level precisely to avoid the occurrence of multiple equilibria. The central bank is then forced to run an inflated

[4] Appendix B provides some suggestive evidence that is indeed the case. Specifically, I document that the variance of forecast errors made by analysts on Italy's macroeconomic outcomes is much larger after the start of the ECB large-scale bond purchase program. This suggests that analysts are not investing as much as before in information production, resulting in much less precise forecasts.

balance sheet to avoid roll-over crises.

Related Literature: This paper is related to three broad literatures: self-fulfilling sovereign defaults ; information production and aggregation in market prices ; and central bank asset purchases.

Self-fulfilling sovereign defaults: The first study of multiplicity of equilibria in models of sovereign debt is due to Calvo (1988). In this model, there is a feedback between interest rates and the debt burden: beliefs about a high probability of default translates into high yields, which makes it optimal next period for the sovereign to indeed default on this debt. Lorenzoni and Werning (2019) expand this approach in a dynamic setting, giving rise to slow-moving crises.[5] My model uses a different type of multiplicity, due to Alesina, Prati and Tabellini (1989), Giavazzi and Pagano (1989), and Cole and Kehoe (2000), called "roll-over" crises in the literature. These models feature two distinct pairs of equilibrium prices and *contemporaneous* default decisions, with multiplicity reminiscent of a bank run.[6] Broner, Erce, Martin and Ventura (2014), Aguiar and Amador (2020), Galli (2021), and Aguiar, Chatterjee, Cole and Stangebye (2022) also offer alternative sources of multiplicity in sovereign debt models.

In all these models featuring multiplicity, a seemingly simple solution (going back to Calvo 1988) to avoid the bad equilibrium is to credibly announce a ceiling on interest rates (for example by abstaining from issuance, or from a lender of last resort mechanism). By eliminating self-fulfilling beliefs about defaults, the bad equilibrium disappears and no action has to be taken in equilibrium. This is the common interpretation attached to the 2012 announcement by Mario Draghi, where he promised to do "whatever it takes" to preserve stability in the Eurozone. Without implementing any transaction, this sole announcement lowered

[5]See also Aguiar, Amador, Farhi and Gopinath (2013), Corsetti and Dedola (2016), Ayres, Navarro, Nicolini and Teles (2018) and Ayres, Navarro, Nicolini and Teles (2023) among others for setups using the Calvo (1988) multiplicity.

[6]See also Chatterjee and Eyigungor (2012), Aguiar, Chatterjee, Cole and Stangebye (2016), Conesa and Kehoe (2017), Roch and Uhlig (2018), and Bocola and Dovis (2019a) for quantitative models featuring roll-over crises.

the spreads of sovereign bonds issued by the distressed European countries.[7] It is nevertheless clear that we now see substantial purchases by the central bank, suggesting this story is incomplete. Furthermore, Lorenzoni and Werning (2019) clarify that in their setup an interest rate ceiling requires a credible commitment to cut spending in this event, which seems implausible. In their words, "in our view there appears to be no easy fix to the multiplicity problem."[8] My paper suggests that actually carrying out asset purchases, under some conditions, can be a possible fix to the problem.

Information production and aggregation: The crucial mechanism in my model comes from endogenous information acquisition, building from the overview provided by Veldkamp (2023). Hellwig and Veldkamp (2009), Hellwig, Kohls and Veldkamp (2012) and Yang (2015) study multiple equilibria in *information choice*. In contrast, I focus on a unique equilibrium in information choice, but study when this equilibrium outcome yields multiple equilibria in bond prices later on.[9] A key insight of my model is that high levels of information acquisition can be excessive and lead to costly (and inefficient) roll-over crises.[10] This is closely related to the idea that crises occur when debt becomes "information-sensitive" (Gorton and Pennacchi 1990, Dang, Gorton and Holmstrom 2009, Gorton and Metrick 2009, Dang, Gorton and Holmström 2012, Gorton and Ordonez 2014, Gorton 2017, Dang, Gorton, Holmström and Ordonez 2017, Gorton and Ordonez 2022). Relatedly, Ahnert and Kakhbod (2017) proposes an amplification mechanism of financial crises based on the information choice of investors, in a global coordination game of regime change. The amplification channels are similar, but my model focuses on information production incentives *ex-ante*, while Ahnert and Kakhbod (2017) studies the production of information that arises after the arrival of bad news. The main contribution is then to show how asset purchases interact with these incentives in

[7] See also Reis (2013) and Corsetti and Dedola (2016) for an exposition of this argument.

[8] See also Chapter 6 of Aguiar and Amador (2019) for discussion on the limitation of mechanisms of lender of last resort.

[9] Hébert and La'O (Forthcoming) provide a general treatment of efficiency and non-fundamental volatility in a class of generalized beauty-contest economies with endogenous information.

[10] Gaballo and Ordonez (2022) show that in incomplete markets environment, the use of information technologies tend to be excessive.

order to avoid multiple equilibria *ex-post*. Another related and complementary theory is provided by Cole, Neuhann and Ordoñez (2022). They focus on *information spillovers*, and show that the structure of price auctions in the primary market leads to strategic complementarities in information acquisition. As result, shocks to default risk in one country may trigger crisis episodes with widespread information acquisition in other risk countries, and falling yields in safe countries.

Central bank asset purchases: This paper considers how information acquisition interacts with asset purchases. The early seminal paper on this policy instrument is by Wallace (1981), who shows that asset purchases by the central bank are irrelevant in a frictionless and closed economy benchmark. This is because taxes need to adjust to compensate for possible losses, therefore exposing debt holders to exactly the same risk. In my paper, debt holders are foreigners and thus not concerned by future tax adjustments.[11] Alternatively, Iovino and Sergeyev (Forthcoming) show that when agents are boundedly-rational, they cannot think through the equilibrium effects of asset purchases on future taxes, so that central bank interventions become relevant. Gaballo and Galli (2022) instead propose a model with heterogeneous beliefs and position bounds. They show that asset purchases are beneficial by crowding out the asset demand of more pessimistic agents, lowering interest rates, debt service and future tax distortions.[12] Bond and Goldstein (2015) study the impact on price informativeness when government intervention influences future cash flows. Brunnermeier, Sockin and Xiong (2022) offer a closely related theory of government intervention in financial markets: leaning against noise traders reduces volatility but at the expense of introducing policy noise into the market. In contrast, in my framework worsening price efficiency is the objective of the policymaker.

[11] This is in the same spirit as the market segmentation literature, where asset purchases have an effect (Curdia and Woodford 2011, Vayanos and Vila 2021, Caballero and Simsek 2021).

[12] An interesting feature of the model of Gaballo and Galli (2022) is that a small amount of purchases is beneficial, but the welfare effects become negative for large programs because they increase the precision of market information in default states. On the contrary, my model argues for large enough asset purchases in order to discontinuously affect welfare by shifting information production below some threshold. The interaction of these effects is outside of the scope of this paper, but suggests that large scale programs might only be warranted in some specific situations.

2 Model

This Section presents a tractable model stripped down to its core in order to focus on the main insights of the paper. Section 5 presents extensions and robustness analyses.

2.1 Setup

The model has three periods: $t \in \{1, 2, 3\}$. At time 1, investors can acquire information about the likelihood of default by the sovereign in the future, and invest in the newly issued bonds. At time 2, the government tries to roll over its debt, and can be subject to roll-over crises and default (Cole and Kehoe 2000) if its debt burden is too high. At time 3, the model ends, and the country repays its remaining debt.

Government: The government consumes only in periods 2 and 3. It has the following utility function:

$$V_2 = u(c_2) + \beta u(c_3) \tag{1}$$

with endowments y_2 and y_3 in periods 2 and 3. In the first period, the government has to finance some expenditures. It can only issue short-term (one period) and non-contingent bonds. Facing bond prices of q_1, the government has a downward-sloping demand for funds: it raises $B_1 q_1 - \phi q_1^2$, meaning that it will have to reimburse $b_1 = B_1 - \phi q_1$ in the next period.[13]

In period $t = 2$, the country can default on its time $t = 1$ issued debt in one of two ways. First, it can exogenously default with probability δ. With probability

[13]This reduced form is taken for tractability only, but does not alter the results. See Appendix D.1 for more details. In Section 5 I instead assume that the country has to finance a fixed set of public programs, and so needs to raise a fixed B_1. This means that it will have to sell B_1/q_1 bonds, i.e. a decreasing function of q_1. The reduced-form $b_1 = B - \phi q_1$ captures the same intuition but keeps a convenient linear formulation, which can be seen as a first-order approximation. Section 5 also studies the case where the government consumes at $t = 1$ and optimally decides how much to issue in period $t = 1$ in order to maximize welfare. The insights developed in this tractable version are robust to these variations. These extensions also allow me to also flesh out the moral hazard implications of asset purchases.

$1-\delta$, it auctions new short-term debt b_2 and then decides whether to repay b_1 after the auction if completed, facing prices $q(b_1, b_2)$ for its newly issued bonds. The value of repayment, conditional on b_2, is:

$$V^R(b_1, b_2) = u(y_2 - b_1 + q(b_1, b_2)b_2) + \beta V_3(b_2) \tag{2}$$

The government repays its debt b_1 if and only if:

$$V^R(b_1, b_2) \geq V_2^D \tag{3}$$

If the country is faced with bond prices of 0 for the entire schedule and does not default on its debt b_1, its value function will be:

$$\underline{V}^R(b_1, 0) = u(y_2 - b_1) + \beta V_3(0) \tag{4}$$

If instead the country decides to default, it will enjoy V_2^D. There will thus be multiple equilibria if and only if:

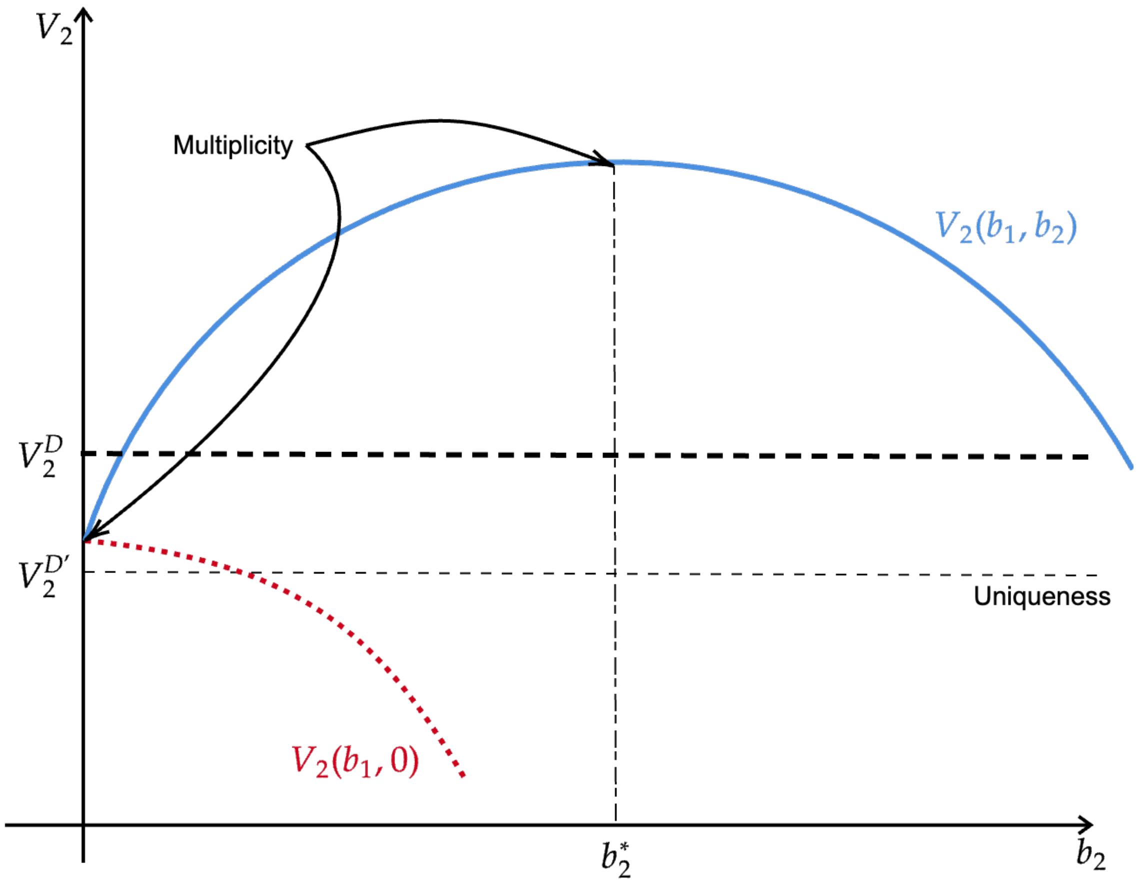
$$\underline{V}^R(b_1, 0) < V_2^D \tag{5}$$

Indeed, when that is the case, a zero price is consistent with individual lenders' optimization: they will not bid a positive price since the government will default.[14] Importantly, this condition depends on b_1, which implies that the equilibrium price at $t = 1$ will be crucial in ensuring whether it is met or not, since $b_1 = B_1 - \phi q_1$. In other words, a lower price at $t = 1$ makes it more likely that there will be a roll-over crisis at $t = 2$. We denote by b_1^* the threshold at which a higher issuance leads to multiple equilibria in the next period. Figure 2 shows graphically when multiplicity appears.

We denote by χ the repayment variable: $\chi = 1$ if the country repays its debt, and $\chi = 0$ if the country defaults. We assume for simplicity that the recovery rate is normalized to 0.

[14] See Aguiar and Amador (2019) for a clear exposition of why the Cole and Kehoe (2000) timing assumptions can lead to multiplicity, and the comparison to the Eaton and Gersovitz (1981) model.

Figure 2: Multiplicity or Uniqueness at $t = 2$. The blue line is the value function when the country repays its debt and issue b_2 in new debt, with a positive price. The red dotted line is the value function when the country repays its debt but cannot issue new debt at a positive price. If this value is below the default value, then the country decides to default and two equilibria are possible. This graph is taken from Aguiar and Amador (2019).



Sunspots: In the event that there is a possibility of a roll-over crisis at $t = 2$, in the sense of Cole and Kehoe (2000), a stochastic process (independent from δ) governs the equilibrium selection. Specifically, with probability λ the equilibrium selected is the crisis one: the price of bonds is 0 for all level of issuance, and the country defaults on its obligations b_1. With probability $1 - \lambda$ the equilibrium selected is the good one, and the country faces positive bond prices and repays b_1. The value of λ is common knowledge to all agents in the economy.

Investors: The country trades its bonds of prices q_1 with mean-variance investors. Holding a quantity b of sovereign bonds yields utility:

$$U = \mathbb{E}[b\chi] - bq_1 - \frac{\sigma^2}{2}\mathbb{V}[b\chi] \tag{6}$$

This naturally implies that an individual investor will choose to hold an amount of sovereign bonds b equal to:

$$b = \frac{\mathbb{E}[\chi] - q_1}{\sigma^2 \mathbb{V}[\chi]} \tag{7}$$

Information Structure: There is a measure 1 of investors, who have a prior over the distribution of δ, with mean $\delta_0 \in [0,1]$ and variance $\mathbb{V}[\delta]$. At the beginning of period 1, investors are heterogeneous in their capacity to acquire information: investor i can pay a fixed cost of $i^2/2\gamma^2$ to acquire information about δ, the exogenous probability of default at $t = 2$.[15] They receive a common signal, and we denote the posterior by $1 - \delta_1 = 1 - \delta_0 + s_1$. The variable s_1 is thus determining by how much investors adjust their expectations or repayment: a positive s_1 means that investors received (good) news that a default is less likely next period. We assume that $s_1 \in \{-s, s\}$. Investors that do not pay the fixed cost are not able to learn that information from prices.[16]

In the case where b_1 is such that there is a unique equilibrium at $t = 2$, we denote by $\mathbb{V}[\delta]$ the prior variance over the exogenous default variable, and by $\mathbb{V}'[\delta]$ the posterior variance when the investor has decided to acquire information. Evidently, $\mathbb{V}'[\delta] < \mathbb{V}[\delta]$.

When there is a possibility of multiple equilibria, we denote the prior variance

[15] Heterogeneous costs of acquiring information are helpful in generating a single equilibrium in information acquisition. For a comprehensive study of complementarities (as well as substitutabilities) in information acquisition, see Veldkamp (2023).

[16] Otherwise, investors would have no incentive to pay this fixed cost (Grossman and Stiglitz 1980). This assumption can be understood as assuming that, by not paying γ, investors do not have sufficient information to understand how prices are formed in equilibrium in order to extract relevant information. Appendix C.2 presents an extension of the model where agents receive noisy signals about the fiscal fundamentals of the sovereign, and can pay a fixed cost in order to be able to learn information from prices. In Appendix C.1, I assume that investors must submit their demand before seeing the price realization. In both cases, the main intuition of the paper are unchanged.

by:

$$\mathbb{V}_\lambda[\delta] = (1-\lambda)(\mathbb{V}[\delta] + (1-\delta_0)^2\lambda) \tag{8}$$

and the posterior variance by $\mathbb{V}'_\lambda[\delta]$.

Acquiring information always lowers the posterior variance used by investors. Because these investors are mean-variance optimizers, this will lower the risk premium they charge for holding sovereign bonds, making it less costly for the sovereign to finance its public expenditures. If this was the only effect of more information, it thus would have a positive effect for the sovereign.

We will often work with precisions τ rather than variances $\mathbb{V}$. The following technical Lemma will prove useful:

Lemma 1. *The benefits of acquiring information with respect to the increased precision are smaller when there are multiple equilibria:*

$$\frac{\tau'_\lambda}{\tau_\lambda} < \frac{\tau'}{\tau} \tag{9}$$

The intuition for this lemma is that information acquisition reduces the variance perceived about the fundamental of the country, but does nothing to address the uncertainty brought by the λ-sunspot.

2.2 Information Acquisition

Lenders decide whether or not to acquire information at the beginning of period 1, taking into account the equilibrium behavior highlighted above. For a given price q, an individual investor i will invest:

$$b_i = \frac{\mathbb{E}[\chi] - q_1}{\sigma^2 \mathbb{V}[\chi]} \tag{10}$$

If we are in a situation where no investor expects a roll-over crisis in the future, an investor who decides not to invest in information acquisition has an expected utility of:

$$\mathbb{E}[U^i|\psi < \psi_s] = \frac{\tau}{2\sigma^2}\mathbb{E}\left[\big(1-\delta_0 - q(s_1)\big)^2\right] - \frac{\tau}{\sigma^2}Cov\big(s_1, q_1(s_1)\big) \tag{11}$$

If instead, investor i decides to acquire information, their expected utility will be:

$$\mathbb{E}[U_\gamma|\psi \leq \psi_s] = \frac{\tau'}{2\sigma^2}\mathbb{E}\left[\left(1-\delta_0+s_1-q(s_1)\right)^2\right] - \frac{i^2}{2\gamma^2} \tag{12}$$

An approximation for small signals can be helpful here. To the first-order when $s \approx 0$, investor i decides to acquire information if and only if:

$$|1-\delta_0-q_1(0)| > \frac{i\sigma}{\gamma\sqrt{\tau'-\tau}} \tag{13}$$

Equation (13) intuitively implies that investors are incentivized to acquire information when: (i) The expected price is away from expectations of fundamentals, (ii) the cost of acquiring information is low, and (iii) acquiring information substantially reduces the uncertainty faced by an investor.

2.3 Equilibrium Description

We start by characterizing prices and quantities for a given ψ share of informed investors. We will then pin down the equilibrium ψ through the individual investor's optimization.[17]

For a given ψ, three different configurations are possible:

1. $q_1(-s;\psi) > (B_1 - b_1^*)\phi^{-1}$
2. $q_1(-s;\psi) \leq (B_1 - b_1^*)\phi^{-1} \leq q_1(s;\psi)$
3. $q_1(s;\psi) < (B_1 - b_1^*)\phi^{-1}$

In the first case, even when informed investors receive negative news about the default probability of the sovereign, the price of bonds is such that the debt burden is below the crises threshold b_1^*. In other words, even in the worst case scenario the country will not face roll-over crises. Investors anticipate this, and thus do not require a premium for bearing risk associated with the sunspot variable λ.

The second case corresponds to the case where the country only faces roll-over crises when investors receive a negative signal. When they receive a positive sig-

[17] In other words, ψ is the marginal investor indifferent between acquiring information or not.

nal, the price of bonds is high enough to sustain a single equilibrium in period $t = 2$.

The third case is the polar opposite of the first one: even when informed investors receive positive news about the default probability of the sovereign, the price of bonds is such that the debt burden is above the crises threshold b_1^*. In other words, even in the best case scenario the country will face roll-over crises.

For clarity, and to really flesh out the mechanism proposed by this paper, the theory will focus on cases 1 and 2, depending on the level of information production ψ.[18]

Unique Equilibrium at $t = 2$ Using the demand functions in (10), when agents do not expect multiple equilibria at $t = 2$, the market clearing condition for any signal s_1 and share of informed agents ψ is:

$$B_1 - \phi q_1(s_1) = \psi\tau' \frac{1 - \delta_0 - s_1 - q_1(s_1)}{\sigma^2} + (1-\psi)\tau \frac{1 - \delta_0 - q_1(s_1)}{\sigma^2} \tag{14}$$

which yields the following equilibrium price:

$$q_1(-s;\psi) = \frac{(1-\delta_0)(\psi\tau' + (1-\psi)\tau) - \psi\tau' s - \sigma^2 B_1}{\psi\tau' + (1-\psi)\tau - \phi\sigma^2} \tag{15}$$

We make two technical assumptions that ensure that we can shift from single to multiple equilibria region as the amount of information production changes.

Assumption 1. *The magnitude of the symmetric signal is such that:*

$$s > \frac{(\tau' - \tau)\sigma^2 (B_1 - \phi(1-\delta_0))}{\tau'(\tau - \phi\sigma^2)} \tag{16}$$

which implies that prices are decreasing in the amount of information acquisition when

[18] Intuitively, between cases 2 and 3 the role of the central bank would be to buy at least enough assets such that in equilibrium, multiple equilibria only arise for a negative signal and not for a positive one, yielding the same qualitative insights. It is however clear that, in the case of a positive signal, the country is better off ex-post with more informed traders. The asset purchases by the central bank would thus not rely on incentives about information production, but other channels already studied in previous work (e.g., Caballero and Simsek 2021), which is why I leave that case aside.

informed investors receive the negative signal:

$$\frac{dq_1(-s;\psi)}{d\psi} < 0 \tag{17}$$

Assumption 1 simply ensures that the price of bonds is decreasing in the number of informed agents when these agents receive negative news. This is not always satisfied because producing information has two effects: first, it does give access to the signal (which exerts a negative force on prices the more agents act on this signal), but it also reduces uncertainty (as seen in the precision term $\tau' > \tau$) which always exerts a positive influence on equilibrium prices by reducing the risk premium required by informed agents to compensate them for bearing default risk. Assumption 1 thus ensures that the first effect is stronger than the second one. Of course, the effect on prices if unambiguous for a positive (or neutral) signal: both effects push in the same direction of making sovereign bonds more expensive.

Assumption 2. *The parameters of the model are such that:*

$$(B_1 - b_1^*)\tau < \phi\big((1-\delta_0)\tau - \sigma^2 b_1^*\big) \tag{18}$$

which implies that, when no investor is informed, prices are high enough to sustain a single equilibrium at $t = 2$:

$$B_1 - \phi q_1(s;0) > b_1^* \tag{19}$$

Assumption 2 ensures that we are in case 1 when $\psi = 0$. Since prices are then decreasing in ψ for a negative signal, we can shift to case 2 when enough agents are informed.

Multiple Equilibria at $t = 2$ Using again the demand functions in (10), when agents do expect multiple equilibria at $t = 2$, the equilibrium price for a negative signal is given by:

$$q_1(-s;\psi) = \frac{(1-\lambda)\Big((1-\delta_0)(\psi\tau'_\lambda + (1-\psi)\tau_\lambda) - \psi\tau'_\lambda s\Big) - \sigma^2 B_1}{\psi\tau'_\lambda + (1-\psi)\tau_\lambda - \phi\sigma^2} \tag{20}$$

which reflects two distinctive features: (i) the payoff is multiplied by $1-\lambda$ since investors anticipate a λ probability of a default because of a roll-over crisis ; and (ii) the precisions used by agents are smaller ($\tau_\lambda < \tau_\lambda$) since sunspots introduce a supplementary source of uncertainty.

Roll-Over Crises Threshold We start by identifying the highest share of informed investors, ψ_s, such that the unique equilibrium at $t=2$ is sustained even in the case of negative news.

Proposition 1. *The information acquisition threshold is:*

$$\psi_s = \frac{\phi(1-\delta_0) - \tau(B_1 - b_1^*) - \phi\sigma^2 b_1^*}{(\tau' - \tau)(B_1 - b_1^*) - \phi(1-\delta_0)(\tau' - \tau) + \tau' s} > 0 \tag{21}$$

1. *If $\psi < \psi_s$, the equilibrium is unique at $t=2$ for any signal realization ;*
2. *If $\psi_s < 1$, there are sunspot (λ) equilibria at $t=2$ whenever $\psi \geq \psi_s$ and the signal realization is $-s$.*

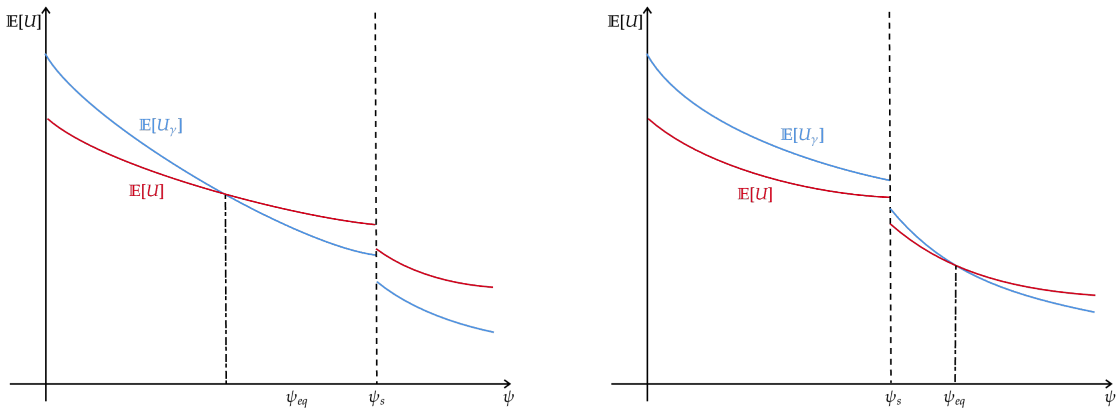
We now only need to pin down the equilibrium ψ in order to fully characterize the equilibrium.

Information Acquisition We now characterize the equilibrium choice of information acquisition. For $\psi \leq \psi_s$ agents anticipate that the equilibrium will be unique next period (such that the only source of default comes from δ), and the expected utility of an investor that does not acquire information is:

$$\mathbb{E}[U^i|\psi < \psi_s] = \frac{\tau}{2\sigma^2}\mathbb{E}\left[\big(1-\delta_0 - q(s_1)\big)^2\right] - \frac{\tau}{\sigma^2}Cov\big(s_1, q_1(s_1)\big) \tag{22}$$

When agents decide to acquire information, they will be able to trade on the signal received, as well as using higher precision $\tau' > \tau$, but will pay the fixed cost:

$$\mathbb{E}[U_\gamma|\psi \leq \psi_s] = \frac{\tau'}{2\sigma^2}\mathbb{E}\left[\big(1-\delta_0 + s_1 - q(s_1)\big)^2\right] - \frac{i^2}{2\gamma^2} \tag{23}$$



(a) Unicity at $t = 2$

(b) Roll-over crisis possible at $t = 2$

Figure 3: Equilibrium determination of ψ.

The equilibrium amount of information production arises (outside of corner solutions) when agents are indifferent between the two options:

$$\mathbb{E}[U_\gamma|\psi \leq \psi_s] = \mathbb{E}[U|\psi \leq \psi_s] \tag{24}$$

We can now formally define the competitive equilibrium.

Definition 1. *A competitive equilibrium at time $t = 1$ is a triplet $\{\psi, q_1(-s), q_1(s)\}$ such that: (i) $q_1(-s)$ and $q_1(s)$ are defined by equation* (14) *; and (ii) equation* (24) *holds.*

The equilibrium determination is depicted in Figure 3. The left panel presents a case where the condition of Proposition 1 is not satisfied. The parameters are such that the crossing of the two expected utility arises at a level ψ below the threshold ψ_s. The right panel presents a case where this condition is satisfied. A share $\psi > \psi_s$ decides to acquire information, such that the equilibrium at $t = 2$ will feature multiplicity in the event that informed investors receive the low signal $s_1 = -s$.

Notice that the expected utility of a trader that decides to acquire information can be written as (with some basic manipulations):

$$\mathbb{E}[U_\gamma|\psi \leq \psi_s] = \frac{\tau'}{2\sigma^2}\mathbb{E}\left[\left(1 - \delta_0 - q(s_1)\right)^2\right] - \frac{\tau'}{\sigma^2}Cov(s_1, q_1(s_1)) + \frac{\tau'}{2\sigma^2}Var(s_1) - \frac{i^2}{2\gamma^2} \tag{25}$$

such that the marginal trader is defined by the equality:

$$\frac{\tau' - \tau}{2\sigma^2}\left(\mathbb{E}\left[\left(1-\delta_0-q(s_1)\right)^2\right] - 2Cov\left(s_1, q_1(s_1)\right)\right) = \frac{\tau'}{2\sigma^2}Var(s_1) + \frac{\psi^2}{2\gamma^2} \tag{26}$$

The right-hand side of this expression is clearly increasing in ψ, while it can be shown that the left-hand side is decreasing in ψ, guaranteeing a unique equilibrium in ψ.

The next proposition simply shows under which condition on γ, the inverse costs of information, we are in the case of the left or right panels.

Proposition 2. *The equilibrium features excessive information production, in the sense that roll-over crises then happen with probability $\lambda/2$, when the costs are acquiring information are small enough. In particular, if:*

$$\gamma > \frac{\psi_s(\psi_s(\tau' - \tau) + \tau - \phi\sigma^2)}{\sigma\sqrt{\tau' - \tau}(B_1 - \phi(1-\delta_0))} \tag{27}$$

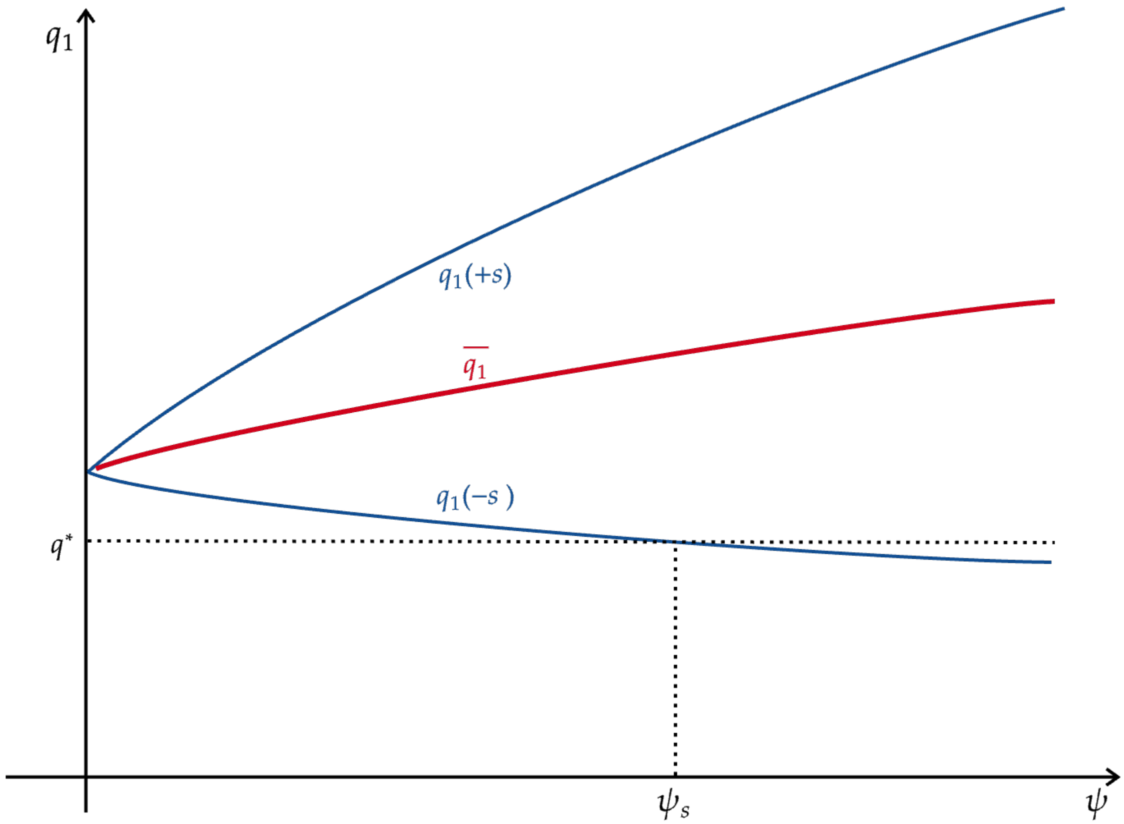
where ψ_s is defined in Proposition 1 by Equation (21), then roll-over crises then happen with probability $\lambda/2$.

2.4 A Graphical Illustration of the Intuition

Most of the intuition of this paper can be contained in a simple graph, as illustrated in Figure 4. Consider a simple asset, for instance a stock. When the share of investors acquiring information, ψ, increases, this is unambiguously beneficial for the average asset price: less uncertainty about future payoffs means that investors on average require a smaller risk premium to hold the asset, leading to a higher price. When the signal realization is negative, the price is slightly decreasing in ψ because of the two competing effects: agents trade with less uncertainty, but more of them trade on negative information about future cash-flows.

When the asset can be subject to runs, however, a new force appears. For a low enough price, here denoted by q^* in Figure 4, multiple equilibria can appear ex-post. Risky sovereign debt is such an asset that can be vulnerable to runs (because of the Cole and Kehoe (2000) mechanism in the present paper, but possibly also

Figure 4: Equilibrium Price of an Asset. This figure plots the equilibrium price on an asset as a function of the share of investors that decide to acquire information. The blue lines are the equilibrium price in the event of two different signal, a positive signal and a negative signal. The red line plots the average equilibrium price. The threshold q^* represents the cutoff at which runs become possible, for example because of roll-over crises as in Cole and Kehoe (2000).



because of runs similar to the Calvo (1988) mechanism). The contribution of this paper is to show that hampering price discovery can then be beneficial in such an environment, and that large-scale asset purchases are one way to achieve this.

3 Asset Purchases

3.1 Supply of Bonds and Information Acquisition Incentives

Why would asset purchases help in this setting? The answer intuitively lies in the expressions for the multiplicity threshold and the equilibrium information choice, equation (21). The crucial element is the presence of B_1: by purchasing bonds directly, the central bank can lower B_1, the amount of bonds that need to be absorbed by investors. This will mechanically alter the ψ threshold at which multiplicity can appear, and importantly also alter the private incentives to acquire information by investors.

This can be seen by looking at how the supply of bonds B_1 impacts the slope of the two expected utility functions that determine the optimal information choice. We can show that:

$$\frac{\frac{d^2 E[U_\gamma|\psi \leq \psi_s]}{dB_1 d\psi}}{\frac{d^2 E[U|\psi \leq \psi_s]}{dB_1 d\psi}} = \frac{\tau'}{\tau} > 1 \tag{28}$$

This expression shows that changing B_1 has a larger impact on the expected utility in the informed case than in the uninformed case, which reduces the incentives to acquire information. This is because of two effects going in the same direction: first, there is less risk to hold in the aggregate. Second, informed investors are more sensitive to supply changes.

The first effect can be understood by assuming (for exposition only) that all investors have to hold the same amount of bonds, b. When that is the case, we can rewrite the difference in expected utility between informed and uninformed investors (not taking into account the costs of acquiring information) as:

$$\Delta \mathbb{E}[U] = b^2 \frac{\sigma^2}{2} \left(\frac{1}{\tau} - \frac{1}{\tau'} \right) \tag{29}$$

This expression makes it clear that, with fewer bonds on their balance sheet, informed investors experience a smaller increase in their expected utility compared to uninformed investors. When investors anticipate that they will hold smaller positions, the increase in utility brought by reducing uncertainty is simply smaller.

The second effect compounds the aggregate one. It can be seen by looking at the average individual positions of investors b^γ for informed investors and b for uninformed:

$$\mathbb{E}[b^\gamma] = \tau' \frac{\sigma^2 (B_1 - \phi(1-\delta_0))}{\psi\tau' + (1-\psi)\tau - \phi\sigma^2} \quad \text{and} \quad \mathbb{E}[b] = \tau \frac{\sigma^2 (B_1 - \phi(1-\delta_0))}{\psi\tau' + (1-\psi)\tau - \phi\sigma^2} \tag{30}$$

These expressions are similar, except for the precision term in front. This implies that the expected position of informed investors is *more sensitive* to changes in B_1, relative to uninformed investors. This effect thus again reduces the gap between the expected utility of an informed investor relative to an informed investor.

3.2 Equilibrium Effects on Information Acquisition

As hinted above, asset purchases by the central bank will affect the incentives for information production by individual agents. Setting aside the issue of multiple equilibria for now, it is clear that this will change the equilibrium risk premium on sovereign bonds, and thus the price at which the sovereign can issue debt. We start by describing how asset purchases change information production in equilibrium.

Proposition 3. *Assume that $\psi < \psi_s$. If the central bank purchases an infinitesimal amount dx_1 of sovereign bonds, the equilibrium share of informed investors decreases:*

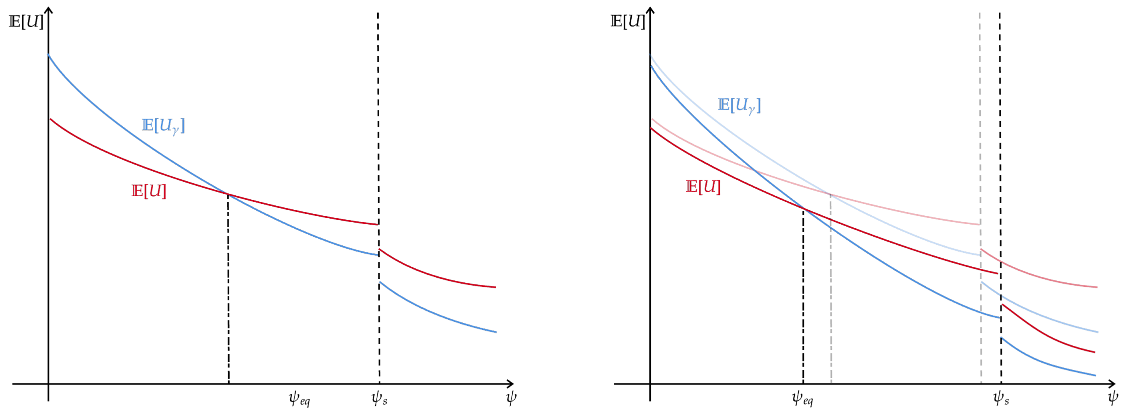
$$\frac{d\psi}{dx_1} < 0 \tag{31}$$

To the first-order in s, it changes according to:

$$d\psi = -\frac{\gamma\sigma\sqrt{\tau' - \tau}}{2(\tau' - \tau)\psi + \tau - \phi\sigma^2} dx_1 \tag{32}$$

Very naturally, we see that the costs of acquiring information are controlling this equilibrium relationship. When γ is large, it is relatively cheap to acquire information. The changes in the equilibrium level of information production are then stronger when asset purchases are implemented. Since the equilibrium features excessive information production (in the sense of putting the sovereign at risk of self-fulfilling default crises) exactly when γ is large enough, asset purchases will be more effective exactly when multiple equilibria are more likely.

The effect of asset purchases by the central bank in this case is shown in the right panel of Figure 5. The intuition coming from changes in supply explains the result of Proposition 3: reducing the supply of bonds lowers the expected utility of informed investors more than the one of uninformed investors, leading to a stronger decline in the blue slope of Figure 5, relative to the red slope. That leads



(a) Unicity at $t = 2$

(b) Asset purchases effect on ψ

Figure 5: Equilibrium determination of ψ and asset purchases left of the threshold ψ_s.

to an intersection for a smaller value of ψ.

A direct implication of this model is thus that asset purchases will lower the amount of information production in equilibrium. Appendix B provides some suggestive evidence that is indeed the case. Specifically, I document that the variance of forecast errors made by analysts on Italy's macroeconomic outcomes is much larger after the start of the ECB large-scale bond purchase program. This suggests that analysts are not investing as much as before in information production, resulting in much less precise forecasts.

This result leads to the following proposition about the effect of asset purchases on the risk premium.

Proposition 4. *Assume that $\psi < \psi_s$. Denote by $r_\sigma = (1 - \delta_0) - \mathbb{E}[q_1]$ the average risk premium on sovereign bonds at $t = 1$. If the central bank purchases an infinitesimal amount dx_1 of sovereign bonds, the effect on the risk premium is ambiguous:*

$$\frac{d \ln r_\sigma}{dx_1} = -\frac{1}{\sigma^2 \big(B_1 - \phi(1 - \delta_0)\big)} \left(\sigma^2 + (\tau' - \tau) r_\sigma \frac{d\psi}{dx_1} \right) \tag{33}$$

which is negative when γ goes to 0 and positive when γ goes to $+\infty$.

This proposition shows that, when roll-over crises do not happen at $t = 2$ because information production is low enough, asset purchases by the central bank have two competing effects: by taking some risk out of private balance sheets, each investor ends up with a smaller position on their portfolio, thus requiring a smaller risk premium, which increases bond prices (Caballero and Simsek 2021 ; Costain et al. 2022). This effect is intuitively quantified by σ^2, the risk aversion of private investors. Concurrently, by weakening the incentives to acquire information, a lower share of investors are informed. This means that, in equilibrium, more investors are trading with higher uncertainty, resulting in lower bond prices. The sum of these two effects determine the equilibrium impact of asset purchases on the risk premium. Proposition 3 also showed how the size of the influence of asset purchases on ψ varies with γ: the equilibrium impact on the risk premium is thus more likely to be positive (and thus counter-productive) when γ is high, i.e. when it is relatively cheap to acquire information.

The following corollary illustrates the same idea, but in terms of the debt burden the sovereign repays at $t = 2$.

Corollary 1. *Assume that $\psi < \psi_s$. If the central bank purchases an infinitesimal amount dx_1 of sovereign bonds, the average debt burden $\bar{b}_1$ changes according to:*

$$\frac{d\bar{b}_1}{dx_1} = \phi \frac{dr_\sigma}{dx_1} \tag{34}$$

or equivalently given the previous proposition:

$$\frac{d\bar{b}_1}{dx_1} = \phi \frac{r_\sigma}{\sigma^2\big(B_1 - \phi(1-\delta_0)\big)} \left(\sigma^2 + (\tau' - \tau) r_\sigma \frac{d\psi}{dx_1}\right) \tag{35}$$

It can be insightful to rewrite this expression to the first-order, as:

$$\frac{d\bar{b}_1}{dx_1} = \frac{\phi r_\sigma}{\sigma\big(B_1 - \phi(1-\delta_0)\big)} \left(\sigma - \frac{\gamma r_\sigma (\tau' - \tau)^{3/2}}{2(\tau' - \tau)\psi + \tau - \phi\sigma^2}\right) \tag{36}$$

to highlight how the trade-off faced by the central bank is dependent on γ. When γ is small, information acquisition is very inelastic. As such, asset purchases have a relatively small effect on the price discovery process, and the balance sheet effect dominates. When γ grows, information acquisition becomes more elastic to the intervention of the central bank, and asset purchases have a greater cost in terms of market efficiency.

3.3 Preventing Rollover Crises

In the presence of multiple equilibria in the next period, the trade-off changes for the central bank. As shown previously, information acquisition makes multiplicity *more* likely. In that case, the adverse effect of asset purchases becomes a strength: by buying assets the central bank discourages information acquisition, and if the asset purchase program is large enough, the share of informed investors becomes low enough such that bond prices are back in the "safe zone."

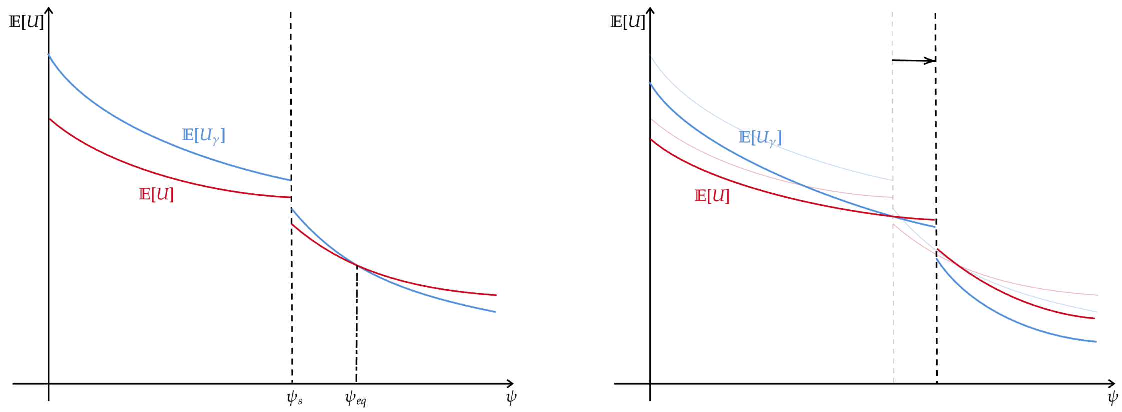
Proposition 5. *If the parameters of the model are such that ψ is higher than ψ_s as defined in (21), i.e. there is enough information acquisition that a roll-over crisis is possible at $t = 2$, then there exists a minimum size x_1^* such that an asset purchase program of size strictly larger than x_1^* ensures that there is a single equilibrium without roll-over crises at $t = 2$. A first-order approximation of an upper-bound to x_1^* is given by:*

$$\bar{x}_1^* = (B_1 - \phi(1 - \delta_0)) - \frac{\psi_s(\psi_s(\tau' - \tau) + \tau - \phi\sigma^2)}{\gamma\sqrt{\tau' - \tau}} \tag{37}$$

As a consequence, the risk premium can be higher on average (more agents are trading with high uncertainty) but rollover crises are avoided. The intuition is shown in Figure 6.

Notice here that this central bank intervention is targeted to avoid multiplicity, but is distinct from the one commonly found in the literature. Starting with the canonical model of Diamond and Dybvig (1983), the mere existence of a credible backstop eliminates the bad equilibrium. This logic also applies to the models of Reis (2013) and Corsetti and Dedola (2016), where the central bank does not have to use any resources on the equilibrium path.[19] By contrast, we do see the ECB routinely buying substantial amounts of sovereign debt, suggesting that this is not the whole story. Proposition 5 shows that, in my model, eliminating future sunspots equilibria requires the central bank to actually carry asset purchases, and

[19] A common interpretation of the "whatever it takes" sequence by Mario Draghi in 2012 is the ECB credibly promising to act as a lender of last resort, eliminating the bad equilibrium. See also Bocola and Dovis (2019b) for a quantitative analysis of this episode.



(a) Rollover crises at $t = 2$

(b) Unicity after asset purchases

Figure 6: Equilibrium effects of asset purchases.

potentially by a substantial amount if ψ is far above the threshold ψ_s.

3.4 Discussion of Mechanisms

The results presented in this section are the product of two key features of the environment. First, information acquisition can lead to self-fulfilling crises in the future. Second, asset purchases by the central bank weaken the incentives of private agent to engage in information acquisition. Government intervention is thus helpful to "make markets more boring" in order to prevent costly runs. The specific channel through which asset purchases affect information acquisition incentives in my model is through the equilibrium supply of bonds, as explained in Section 3.1. This is not, however, crucial for my results. The main insight of this paper is to argue that government intervention (by virtue of being a large player) can be beneficial by precisely impairing the regular functioning of financial markets when assets can be subject to self-fulfilling runs. Other channels weakening information acquisition could be equally considered: for example in Brunnermeier et al. (2022), agents prefer to acquire information about the noise induced by policy actions rather than fundamentals.

4 Welfare, Information, and Asset Purchases

This Section applies the insights of the previous model to understand the welfare benefits (or costs) of asset purchase programs in this setup.[20] Start with the value function of the government at $t = 2$, in the eventuality that no default occurs. Given a stock of debt b_1 to repay, the value function is:

$$V^R(b_1, b_2) = \ln(y_2 - b_1 + \beta b_2) + \beta \ln(y_3 - b_2) \tag{38}$$

And optimal consumption smoothing for the government leads to:

$$V^R(b_1) = (1+\beta)\ln\left(\frac{y_2 - b_1 + \beta y_3}{1+\beta}\right) = (1+\beta)\ln\left(\frac{Y_2 - b_1}{1+\beta}\right) \tag{39}$$

where $Y_2 = y_2 + \beta y_3$ is the present value of endowments. If the government defaults, it has to consume in autarky and suffers output losses such that its value function is:

$$V_2^D = \ln(\underline{y_2}) + \beta \ln(y_3) = V^D \tag{40}$$

Focusing first on the case where the government does not default and there are no rollover crises ($\psi < \psi_s$), we can show that government welfare depends on ψ according to:

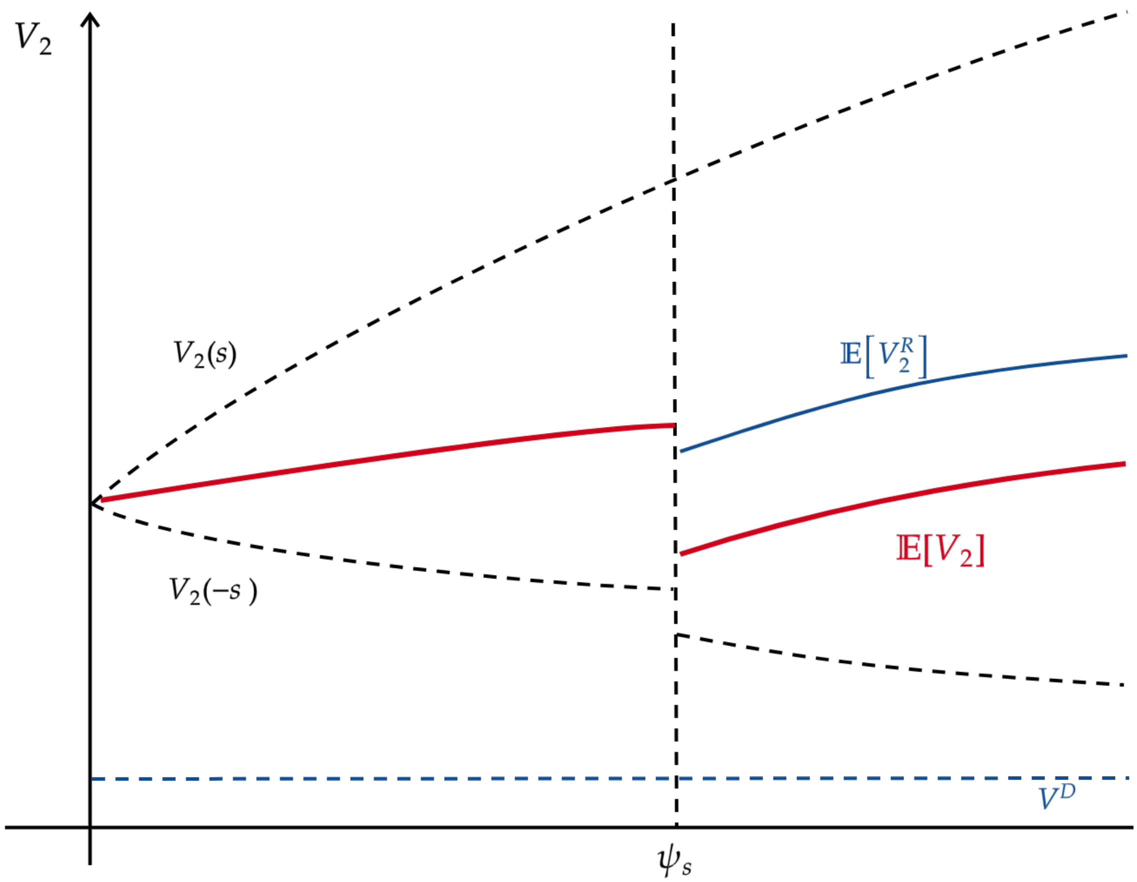
$$\begin{aligned}\frac{dV^R}{d\psi} = & \underbrace{\frac{(\tau' - \tau)\sigma^2(B_1 - \phi(1-\delta_0))}{2(\psi\tau' + (1-\psi)\tau - \phi\sigma^2)^2}\left(\frac{1}{Y_2 - B_1 + \phi q_1(s)} + \frac{1}{Y_2 - B_1 + \phi q_1(-s)}\right)}_{\text{risk premium term} \quad >0} \\ & + \underbrace{\frac{\tau'\tau}{2(\psi\tau' + (1-\psi)\tau - \phi\sigma^2)^2}\left(\frac{1}{Y_2 - B_1 + \phi q_1(s)} - \frac{1}{Y_2 - B_1 + \phi q_1(-s)}\right)}_{\text{value function concavity term} \quad <0}\end{aligned} \tag{41}$$

This expression is always positive to the first order in s: the concavity term goes to 0 as it comes from the risk-aversion of the sovereign, while the first term is strictly positive.

[20] For simplicity, this section only considers the welfare of the government and not of the central bank that is carrying the purchases. In Section 5.2, I also take into account that the central bank dislikes being exposed to the risk of default.

The above logic also naturally goes through when $\psi > \psi_s$, but with different variance terms since bondholders now require a higher compensation for being exposed to rollover risk. The difference, however, is a discontinuity of the value function at ψ_s when bondholders receive a negative signal while there is no such discontinuity in the positive signal case. This discontinuity is because of the the discrete jump in the risk premium required by bondholders, not directly because a bad equilibria might happen. This is represented in Figure 7.

Figure 7: Sovereign's value function at $t = 2$. The blue lines represent how the value function changes with the amount of information acquisition realized at $t = 1$, for a positive and a negative signal. The red line is the expectation over the signal realization, and over the realization of the sunspot λ. The blue line is the expected value in the case where the sunspot coordinates an equilibrium with repayment. The blue dotted line is the value upon defaulting. The discontinuity arises for a negative signal because of a rollover crisis possibility when $\psi > \psi_s$.



Once we combine these results with the previous observation that asset purchases reduce incentives for information acquisition, this graph contains the intuition for the three main insights of this paper. First, asset purchases can be harmful when then is no rollover crises risk. Second, asset purchases can improve welfare by reducing information acquisition in order to fully avoid rollover crises later on. Third, asset purchases need to have a minimal size to be effective, they can otherwise reduce welfare even if there is a risk of a rollover crisis. I now go over these results in turn.

Proposition 6. *When $\psi < \psi_s$, the effect of small asset purchases on the sovereign's welfare is ambiguous, and always negative for large enough costs of acquiring information γ. To the first-order, the average welfare effect is given by:*

$$\frac{d\mathbb{E}V}{dx_1} = \frac{\phi(1+\beta)}{\bar{c}_2} \frac{r_\sigma}{\sigma^2\big(B_1 - \phi(1-\delta_0)\big)} \left(\sigma^2 + (\tau' - \tau) r_\sigma \frac{d\psi}{dx_1}\right) \tag{42}$$

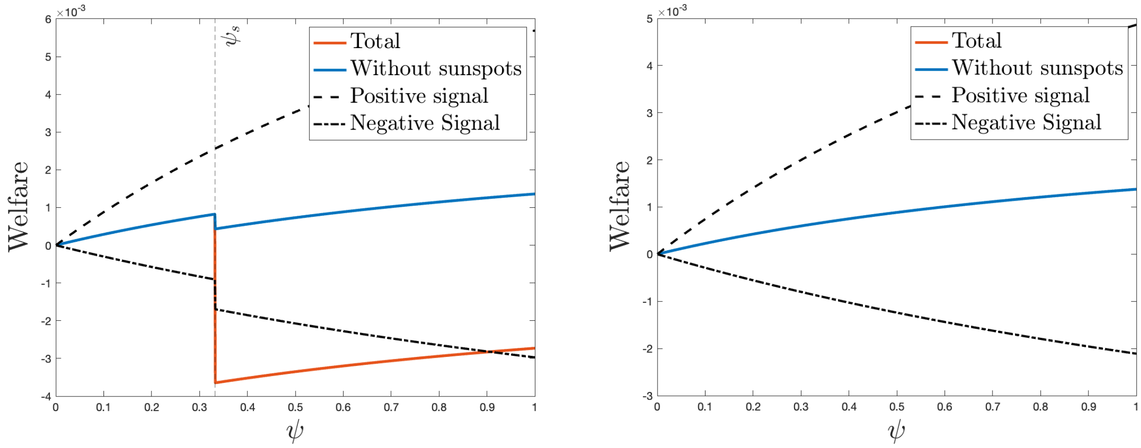
This proposition simply extends the result of Proposition 4 to the welfare perspective of the sovereign. We now focus on the case where $\phi > \phi_s$ without asset purchases. We also assume that V^D is low enough (and λ is non-zero), such that the welfare of the government is higher without rollover crises, whatever the level of information acquisition ψ is.[21] In that situation, asset purchases are valuable to reduce information acquisition and ensure that roll-over crises are impossible.

Proposition 7. *The welfare of the sovereign is strictly higher when the central bank implements asset purchases of the size described in Proposition 5.*

Finally, this minimum size is crucial for the welfare results. Otherwise, asset purchases have an ambiguous effect and can be welfare reducing. This is simply for the same reason as to the left of the threshold: in both cases, asset purchases have a negative effect on information acquisition. For small asset purchases, information acquisition is reduced but the probability of a roll-over crises stays the same.

Proposition 8. *The effect of small asset purchases $x_1 < x_1^*$ when $\psi > \psi_s$ is ambiguous and can be welfare reducing.*

[21] It is entirely possible to be in a situation when the level of welfare is higher when $\phi = 1$ for instance. This is typically the case when ψ_s is very close to 0. In that case, there is a second threshold value $\psi_V > \psi_s$ such that asset purchases are not welfare improving when $\psi > \psi_V$. See Figure 14 in Appendix D.



(a) High B_1 (b) Low B_1

Figure 8: Welfare at $t = 2$ when $b_1 = B_1/q_1$.

5 Extensions and Robustness

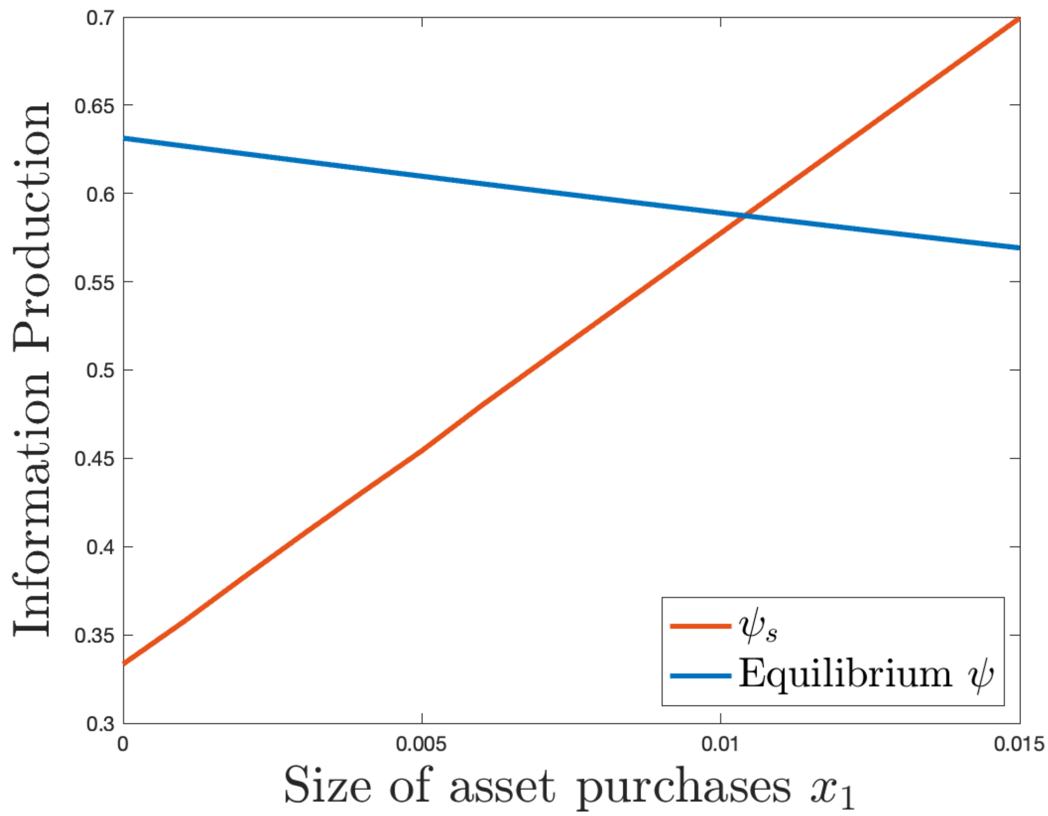
5.1 Fixed Expenditures at $t = 1$

The stylized model presented in the core of the paper assumed a convenient formulation for the supply of bonds, $b_1 = B_1 - \phi q_1$, in order to achieve analytical solutions. This section verifies that all the insights presented above go through when we instead assume that the sovereign must finance a fixed amount of expenditures B_1, such that the supply of bonds is given by $b_1 = B_1/q_1$. The equivalent of Figure 7 is presented in Figure 8. While closed-form solutions are not available anymore, the insights are similar.[22] Figure 9 additionally shows how asset purchases distort the equilibrium choice of information ψ, and the threshold level ψ_s.

Figure 10 then plots the welfare effects of asset purchases on the sovereign, and illustrates the insights of Propositions 6, 7 and 8. The presence of roll-over crises create a positive jump in welfare if the central bank, by purchasing enough sovereign bonds, is able to push the equilibrium ψ below the thresholds ψ_s.

[22]This was expected, since in both cases there is a downward-supply of bonds.

Figure 9: Equilibrium and Threshold Amount of Information Production. The blue line gives the equilibrium share of agents that purchase information (by paying γ) as a function of asset purchases by the central bank, x_1. The red line corresponds to the threshold level psi_s such that, if in equilibrium $\psi > \psi_s$, then roll-over crises are possible when informed traders receive the negative signal. These two lines cross at x_1^*, the minimum size of asset purchases necessary to avoid roll-over crises.

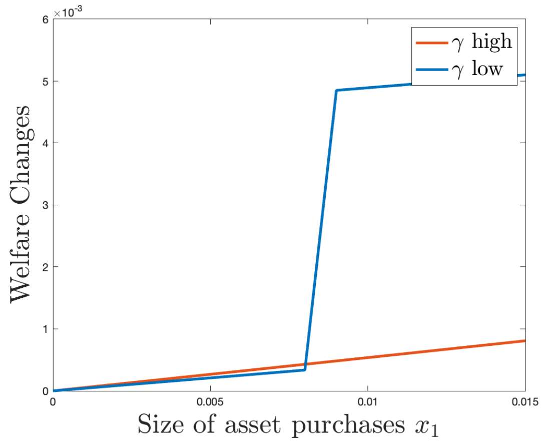


5.2 Costly Asset Purchases

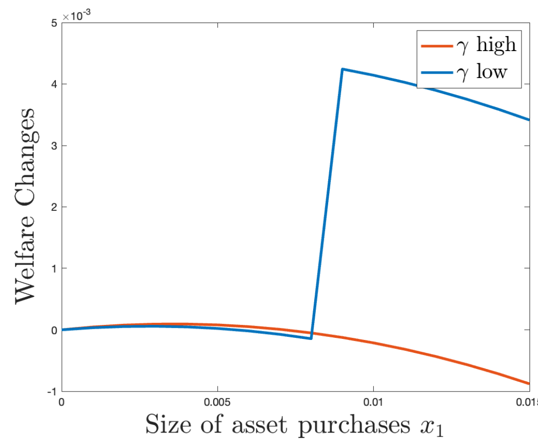
The analysis in the main framework of the paper assumed away the costs of running a large balance sheet for the central bank, in order to focus on its welfare-improving side. Because of the exogenous probability of default, δ, the central bank does expose itself to losses by carrying asset purchases. Even if the central bank is able to fully eliminate the roll-over risk, the asset is still risky.[23]

This extension thus simply assumes that the central bank is itself risk-averse, with a coefficient of risk-aversion σ^2_{CB}, in order to incorporate the costs of expanding its balance sheet with risky assets. By purchasing x sovereign bonds, the central

[23] Note that this is a necessary feature of the model. If debt is entirely risk-free, there are no incentives to gather information for private investors.



(a) Costless asset purchases.



(b) Costly asset purchases

Figure 10: Welfare Effects of Asset Purchases for High and Low Costs of Acquiring Information

bank then simply incurs a cost:

$$\mathcal{L}_{CB} = \frac{1}{2}\sigma_{CB}^2 x^2 \tag{43}$$

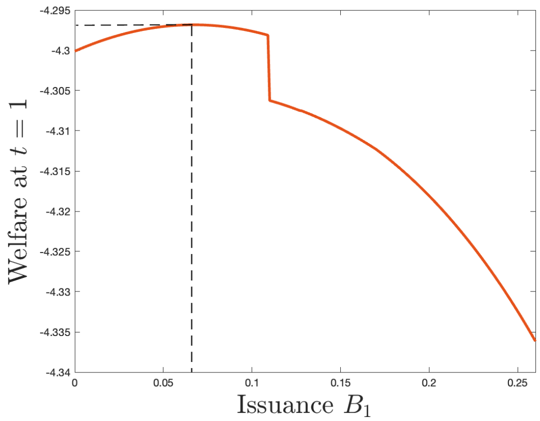
The right panel of Figure 10 illustrates the same results as before but further assumes this quadratic cost of using its balance-sheet.[24] This makes small asset purchases detrimental for welfare, but the discontinuity that arises by avoiding roll-over crises entirely makes up for that cost, again only when asset purchases are large enough in scale.

5.3 Optimal Debt Issuance at $t = 1$

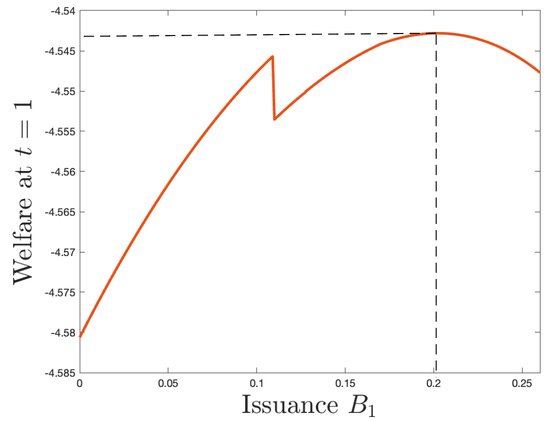
Both the main framework used to present the results and the previous extension assumed a mechanical relation between the price q_1 and the level of issuance by the sovereign b_1. It is natural to believe, however, that a country faced with such prices at issuance would reduce its indebtedness in order to avoid the occurrence of roll-over crises. This extension shows that this is not necessarily the case.

We thus assume for this part that the government also consumes at time $t = 1$,

[24]This cost can also be interpreted as a political cost.



(a) High y_1



(b) Low y_1.

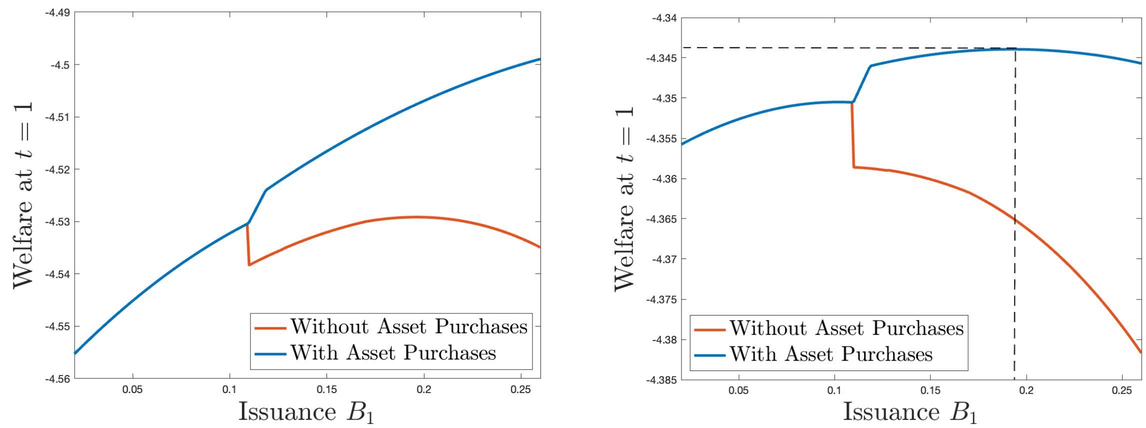
Figure 11: Sovereign's Welfare for different levels of issuance B_1. On the left panel, the level of issuance that maximizes welfare yields in equilibrium a share of informed investors $\psi > \psi_s$, this leading to multiple equilibria when investors receive the negative signal. On the right panel, welfare is higher for issuance levels that avoid the possibility of roll-over crises.

and maximizes life-time utility:

$$V_1 = u(c_1) + u(c_2) + \beta u(c_3) \tag{44}$$

where u is a CRRA utility function. The sovereign received an endowment y_1 in period 1, and thus consumes $c_1 = y_1 + q_1 b_1$. The government decides how much to issue — optimally — understanding how equilibrium prices are formed. In other words, the government realizes that for some levels of issuance, information acquisition in equilibrium is high enough such that the country will face roll-over crises in the future in some states of the world. Figure 11 plots lifetime welfare levels at $t = 1$ along different levels of issuance $B_1 = b_1 q_1$. This figure shows clearly how, above some level of issuance, expected welfare drops discontinuously because of self-fulfilling default at $t = 2$.

Figure 11 illustrates that the government must balance the costs of facing possible roll-over crises in the future with the benefits of consumption smoothing. In particular, the two panels show that, for high levels of endowment y_1 in period 1, the consumption smoothing motive is muted, thus avoiding roll-over crises is optimal. When y_1 becomes low enough, consumption smoothing becomes important enough for the sovereign to accept being exposed to self-fulfilling defaults later on.



(a) Low y_1.　　(b) High y_1

Figure 12: Sovereign's Welfare for different levels of issuance B_1, taking into account that the central bank can implement asset purchases.

5.4 Anticipated Asset Purchases and Moral Hazard

The previous section shows that, in this setup, sovereigns can willingly expose themselves to possible roll-over crises. Since we also showed that it is optimal for the central bank to step in and purchase large amounts of sovereign bonds, this naturally leads to the question of moral hazard. If the sovereign realizes that the central bank will intervene, what is the optimal decision in terms of debt issuance?

Figure 12 shows the equilibrium issuance level with and without asset purchases, again for different levels of endowment y_1. The left panel illustrates an expected result: in the case where the government exposes itself to roll-over crises irrespective of asset purchases, the anticipated action of the central bank leads the sovereign to issue even more. Indeed, the sovereign now faces higher bond prices which makes it more attractive to borrow.

The most interesting result is depicted on the right panel of figure 12. In the case where y_1 is high enough so that the sovereign would normally not expose itself to self-fulfilling roll-over crises, the anticipation of asset purchases leads the sovereign to issue more bonds: so much more that, without asset purchase, this level would lead to multiple equilibria at $t = 2$.

5.5 Uncertainty over the Scale of Purchases

Appendix E studies the case where the market is uncertain about the precise size of x_1 to be implemented by the central bank. I show that the introduction of uncertainty over x_1 increases the expected utility of informed traders while keeping it unchanged for uninformed traders. As a result, this pushes for more information acquisition, the opposite of the effect sought by the central bank. In conclusion, reducing the uncertainty over the precise scale of future asset purchases expected by the market is desirable for central bankers that seek to reduce information acquisition.

6 Conclusion

This paper proposes a new theory of large-scale asset purchases of sovereign bonds by central bank. The key innovation of the model is to allow for endogenous information production. While information acquisition reduces the uncertainty faced by investors and thus lowers risk premia, it makes bond prices more sensitive to new information. When a large number of investors acquire information, bond prices fall substantially after negative information, which can precipitates the sovereign into roll-over crises later on.

Asset purchases by the central bank can play a welfare-enhancing role in this setup. By transferring risks away from private balance sheets, the central bank discourages information acquisition. While this hampers price informativeness, this paper argues that this could be a feature rather than a bug of such large scale programs. By implementing (potentially substantial) purchases, the central bank can avoid the occurrence of roll-over crises in the event of bad news, generating large welfare gains. Finally, when the sovereign expects the central bank to carry such programs, it leads to excessive indebtedness, forcing the central bank to run an inflated balance sheet to avoid roll-over crises.

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Appendices

A Derivations

A.1 Proof of Lemma 1

This is simply coming from basic manipulations. First we have:

$$\frac{1}{\tau'_\lambda \tau} = (1-\lambda)\left(\frac{1}{\tau'\tau} + \lambda(1-\delta_0^2)\frac{1}{\tau}\right) \tag{A.1}$$

and

$$\frac{1}{\tau_\lambda \tau'} = (1-\lambda)\left(\frac{1}{\tau'\tau} + \lambda(1-\delta_0^2)\frac{1}{\tau'}\right) \tag{A.2}$$

so that:

$$\frac{1}{\tau'_\lambda \tau} - \frac{1}{\tau_\lambda \tau'} = (1-\lambda)\lambda(1-\delta_0^2)\left(\frac{1}{\tau} - \frac{1}{\tau'}\right) \tag{A.3}$$

$$\implies \frac{1}{\tau'_\lambda \tau} - \frac{1}{\tau_\lambda \tau'} > 0 \tag{A.4}$$

since $\tau < \tau'$. Hence:

$$\frac{\tau'_\lambda}{\tau_\lambda} < \frac{\tau'}{\tau} \tag{A.5}$$

A.2 Assumption 1

We look at the equilibrium price when investors do not expect sunspot equilibria at $t = 2$, and a share ψ of them acquired information. The market clearing condition when informed investors receive the negative signal is then:

$$B_1 - \phi q_1 = \psi\tau'\frac{1-\delta_0-s-q_1}{\sigma^2} + (1-\psi)\tau\frac{1-\delta_0-q_1}{\sigma^2} \tag{A.6}$$

which yields the following equilibrium price:

$$q_1(-s;\psi) = \frac{(1-\delta_0)(\psi\tau' + (1-\psi)\tau) - \psi\tau' s - \sigma^2 B_1}{\psi\tau' + (1-\psi)\tau - \phi\sigma^2} \tag{A.7}$$

We are interested in the derivative with respect to ψ:

$$\frac{dq_1(-s;\psi)}{d\psi} = \frac{((1-\delta_0)(\tau'-\tau)-\tau's)(\psi\tau'+(1-\psi)\tau-\phi\sigma^2)}{(\psi\tau'+(1-\psi)\tau-\phi\sigma^2)^2} - \frac{(\tau'-\tau)(1-\delta_0)(\psi\tau'+(1-\psi)\tau)-\psi\tau's-\sigma^2 B_1)}{(\psi\tau'+(1-\psi)\tau-\phi\sigma^2)^2} \tag{A.8}$$

Simplifying:

$$\frac{dq_1(-s;\psi)}{d\psi} = \frac{(\tau'-\tau)(-(1-\delta_0)\phi\sigma^2+\sigma^2 B_1)}{(\psi\tau'+(1-\psi)\tau-\phi\sigma^2)^2} - \frac{\tau's(\psi\tau'+(1-\psi)\tau-\phi\sigma^2-\psi(\tau'-\tau))}{(\psi\tau'+(1-\psi)\tau-\phi\sigma^2)^2} \tag{A.9}$$

$$\frac{dq_1(-s;\psi)}{d\psi} = \frac{(\tau'-\tau)\sigma^2(B_1-\phi(1-\delta))}{(\psi\tau'+(1-\psi)\tau-\phi\sigma^2)^2} - \frac{\tau's(\tau-\phi\sigma^2)}{(\psi\tau'+(1-\psi)\tau-\phi\sigma^2)^2} \tag{A.10}$$

Since $B_1-\phi(1-\delta_0) > 0$ (it is exactly equal to b_1, the debt burden when bonds are priced as if they were risk-free and with neutral information), this derivative is negative if and only if:

$$(\tau'-\tau)\sigma^2(B_1-\phi(1-\delta)) < \tau's((\tau-\phi\sigma^2) \tag{A.11}$$

which implies the expression given in Assumption 1:

$$s > \frac{(\tau'-\tau)\sigma^2(B_1-\phi(1-\delta))}{\tau'(\tau-\phi\sigma^2)} \tag{A.12}$$

A.3 Assumption 2

The price of bonds when no investors are informed is given by:

$$q_1(s;0) = \frac{(1-\delta_0)\tau-\sigma^2 B_1}{\tau-\phi\sigma^2} \tag{A.13}$$

where the signal realization is irrelevant since no investor can trade on it. Since the supply of bonds is equal to $B_1 - \phi q_1$, the country will be below the roll-over crisis threshold if and only if:

$$B_1 - \phi \frac{(1-\delta_0)\tau - \sigma^2 B_1}{\tau - \phi\sigma^2} < b_1^* \tag{A.14}$$

which is equivalent to:

$$(B_1 - b_1^*)(\tau - \phi\sigma^2) < \phi((1-\delta_0)\tau - \sigma^2 B_1) \tag{A.15}$$

and finally:

$$(B_1 - b_1^*)\tau < \phi\big((1-\delta_0)\tau - \sigma^2 b_1^*\big) \tag{A.16}$$

A.4 Proof of Proposition 1

A single equilibrium at $t = 2$ can be sustained when:

$$B_1 - \phi \frac{(1-\delta_0)(\psi\tau' + (1-\psi)\tau) - \psi\tau' s - \sigma^2 B_1}{\psi\tau' + (1-\psi)\tau - \phi\sigma^2} < b_1^* \tag{A.17}$$

The threshold is then defined when this is an equality, which yields:

$$B_1 - \phi \frac{(1-\delta_0)(\psi_s\tau' + (1-\psi_s)\tau) - \psi_s\tau' s - \sigma^2 B_1}{\psi_s\tau' + (1-\psi_s)\tau - \phi\sigma^2} = b_1^* \tag{A.18}$$

which gives:

$$\psi_s = \frac{\phi(1-\delta_0) - \tau(B_1 - b_1^*) - \phi\sigma^2 b_1^*}{(\tau' - \tau)(B_1 - b_1^*) - \phi(1-\delta_0)(\tau' - \tau) + \tau' s} \tag{A.19}$$

A.5 Proof of Proposition 2

The equilibrium level of ψ is above the threshold when:

$$\mathbb{E}[U|\psi \to \psi_s^-] < \mathbb{E}[U_\gamma|\psi \to \psi_s^-] \tag{A.20}$$

which means that right at the threshold, an individual has higher expected utility by paying the fixed cost to acquire information.

Start from the expressions of expected utility for both types of traders:

$$\mathbb{E}[U_\gamma|\psi \leq \psi_s] = \frac{\tau'}{2\sigma^2}\mathbb{E}\left[\big(1-\delta_0-q(s_1)\big)^2\right] - \frac{\tau'}{\sigma^2}Cov\big(s_1, q_1(s_1)\big) + \frac{\tau'}{2\sigma^2}\mathbb{V}(s_1) - \frac{i^2}{2\gamma^2} \quad \text{(A.21)}$$

and

$$\mathbb{E}[U^i|\psi < \psi_s] = \frac{\tau}{2\sigma^2}\mathbb{E}\left[\big(1-\delta_0-q(s_1)\big)^2\right] - \frac{\tau}{\sigma^2}Cov\big(s_1, q_1(s_1)\big) \quad \text{(A.22)}$$

Call $\Delta\mathbb{E}$ the difference between expected utilities (without incorporating the fixed cost of information acquisition), to write:

$$\Delta\mathbb{E} = \frac{\tau'-\tau}{2\sigma^2}\left[\mathbb{E}\left[\big(1-\delta_0-q(s_1)\big)^2\right] - 2Cov\big(s_1, q_1(s_1)\big)\right] + \frac{\tau'}{2\sigma^2}\mathbb{V}(s_1) \quad \text{(A.23)}$$

The expectation term inside the first parentheses can be simplified:

$$\mathbb{E}\left[\big(1-\delta_0-q(s_1)\big)^2\right] = \frac{1}{2}\left[(1-\delta_0-q(s))^2 + (1-\delta_0-q(-s))^2\right] \quad \text{(A.24)}$$

and write the price as:

$$q(s_1) = \bar{q} + \Psi s \quad \text{(A.25)}$$

where:

$$\bar{q} = \frac{(1-\delta_0)(\psi\tau' + (1-\psi)\tau) - \sigma^2 B_1}{\psi\tau' + (1-\psi)\tau - \phi\sigma^2} \quad \text{and} \quad \Psi = \frac{\psi\tau'}{\psi\tau' + (1-\psi)\tau - \phi\sigma^2} \quad \text{(A.26)}$$

This helps simplifying the squared expressions since:

$$\frac{1}{2}\left[(1-\delta_0-q(s))^2 + (1-\delta_0-q(-s))^2\right] = \frac{1}{2}\left[(1-\delta_0-\bar{q}-\Psi s)^2 + (1-\delta_0-\bar{q}+\Psi s)^2\right] \quad \text{(A.27)}$$

$$= (1-\delta_0-\bar{q})^2 + \Psi^2 s^2 \quad \text{(A.28)}$$

Notice also that:

$$Cov\big(s_1, q_1(s_1)\big) = \Psi \mathbb{V}(s_1) = \Psi s^2 \tag{A.29}$$

so that we end up with:

$$\Delta \mathbb{E} = \frac{\tau' - \tau}{2\sigma^2} \left[(1 - \delta_0 - \bar{q})^2 + \Psi^2 s^2 - 2\Psi s^2 \right] + \frac{\tau'}{2\sigma^2} s^2 \tag{A.30}$$

Putting together the s^2 terms, we have an expression proportional to:

$$(\tau' - \tau)\Psi(\Psi - 2) + \tau' \tag{A.31}$$

But the $\Psi(\Psi - 2)$ is a quadratic function, so its minimum is when $\Psi = 1$. At this point, this expression becomes $-(\tau' - \tau) + \tau' = \tau > 0$. This expression is thus always strictly positive. This implies that if the following condition is satisfied:

$$\frac{\tau' - \tau}{2\sigma^2}(1 - \delta_0 - \bar{q}(\psi_s))^2 \geq \frac{\psi_s^2}{2\gamma^2} \tag{A.32}$$

then the equilibrium ψ is guaranteed to be above ψ_s. Because we also have:

$$1 - \delta_0 - \bar{q}(\psi_s) = \frac{\sigma^2 (B_1 - \phi(1 - \delta_0)}{\psi\tau' + (1 - \psi)\tau - \phi\sigma^2} \tag{A.33}$$

then this condition becomes:

$$\gamma > \frac{\psi_s(\psi_s(\tau' - \tau) + \tau - \phi\sigma^2)}{\sigma\sqrt{\tau' - \tau}(B_1 - \phi(1 - \delta_0))} \tag{A.34}$$

A.6 Proof of Proposition 3

Start by inspecting the expected utility terms:

$$\mathbb{E}[U^i | \psi < \psi_s] = \frac{\tau}{2\sigma^2} \mathbb{E}\left[\big(1 - \delta_0 - q(s_1)\big)^2 \right] - \frac{\tau}{\sigma^2} Cov\big(s_1, q_1(s_1)\big) \tag{A.35}$$

and

$$\mathbb{E}[U_\gamma | \psi \leq \psi_s] = \frac{\tau'}{2\sigma^2} \mathbb{E}\left[\big(1 - \delta_0 + s_1 - q(s_1)\big)^2 \right] - \frac{i^2}{2\gamma^2} \tag{A.36}$$

Writing once again $q(s_1) = \bar{q} + \Psi s_1$, and realizing that Ψ does not depend on B_1, looking only at changes in $\bar{q}$ (caused by the asset purchase dx_1) we get:

$$\frac{d\mathbb{E}[U^i|\psi < \psi_s]}{d\bar{q}} = -\frac{\tau}{\sigma^2}(1 - \delta_0 - \bar{q}) \tag{A.37}$$

while

$$\frac{d\mathbb{E}[U^i_\gamma|\psi < \psi_s]}{d\bar{q}} = -\frac{\tau'}{\sigma^2}(1 - \delta_0 - \bar{q}) \tag{A.38}$$

This expression makes clear that the expected utility of informed traders falls more than the expected utility of uninformed traders, thus directly leading to a decrease of ψ in equilibrium since both expected utility are decreasing in the equilibrium ψ.

With an infinitesimal amount of asset purchases dx_1 we can write:

$$\bar{q}_1(dx, d\psi) = \frac{(1-\delta_0)((\psi + d\psi)(\tau' - \tau) + \tau) - \sigma^2(B_1 - dx)}{(\psi + d\psi)(\tau' - \tau) + \tau - \phi\sigma^2} \tag{A.39}$$

The first-order development is thus given by:

$$\begin{aligned}\bar{q}_1(dx, d\psi) = \bar{q}_1 + d\psi\frac{(1-\delta_0)(\tau'-\tau)}{\psi(\tau'-\tau) + \tau - \phi\sigma^2} + dx\frac{\sigma^2}{\psi(\tau'-\tau) + \tau - \phi\sigma^2} \\ - d\psi(\tau'-\tau)\frac{\bar{q}_1}{\psi(\tau'-\tau) + \tau - \phi\sigma^2}\end{aligned} \tag{A.40}$$

Now notice that the equilibrium on information production requires that, to the first order in s (see Appendix A.5):

$$\gamma = \frac{\psi(\psi(\tau'-\tau) + \tau - \phi\sigma^2)}{\sigma\sqrt{\tau'-\tau}(B_1 - \phi(1-\delta_0))} \tag{A.41}$$

Which implies that, with asset purchases:

$$\gamma = \frac{(\psi + d\psi)((\psi + d\psi)(\tau'-\tau) + \tau - \phi\sigma^2)}{\sigma\sqrt{\tau'-\tau}(B_1 - dx_1 - \phi(1-\delta_0))} \tag{A.42}$$

Which yields:

$$-\gamma\sigma\sqrt{\tau'-\tau}dx_1 = d\psi(2\psi(\tau'-\tau) + \tau - \phi\sigma^2) \tag{A.43}$$

and hence:

$$d\psi = -\frac{\gamma\sigma\sqrt{\tau' - \tau}}{(2\psi(\tau' - \tau) + \tau - \phi\sigma^2)} dx_1 \tag{A.44}$$

$\square$

Without taking the first-order approximation, the expression is still unambiguously negative (which is the important prediction of the model) but more involved:

$$\begin{aligned} -dx_1 2\sigma^2(1 - \delta_0 - \bar{q}_1) = d\psi \Bigg[\mathbb{V}(s) \frac{\tau'(\tau' - \phi\sigma^2)}{\psi(\tau' - \tau) + \tau - \phi\sigma^2} \\ + 2(\tau' - \tau)(1 - \delta_0 - \bar{q}_1) + \frac{\psi\sigma^2}{\gamma^2(\tau' - \tau)}(\psi(\tau' - \tau) + \tau - \phi\sigma^2) \Bigg] \end{aligned} \tag{A.45}$$

so that:

$$\frac{d\psi}{dx_1} < 0 \tag{A.46}$$

A.7 Proof of Proposition 4

Use the fact that:

$$r_\sigma = 1 - \delta_0 - \bar{q} \tag{A.47}$$

which immediately implies that:

$$\frac{dr_\sigma}{dx_1} = -\frac{\partial \bar{q}}{\partial x_1} - \frac{\partial \bar{q}}{\partial \psi} \frac{d\psi}{dx_1} \tag{A.48}$$

Then, going back to the first-order development from the previous proof after asset purchases:

$$\begin{aligned} \bar{q}_1(dx, d\psi) = \bar{q}_1 + d\psi \frac{(1 - \delta_0)(\tau' - \tau)}{\psi(\tau' - \tau) + \tau - \phi\sigma^2} + dx \frac{\sigma^2}{\psi(\tau' - \tau) + \tau - \phi\sigma^2} \\ - d\psi(\tau' - \tau) \frac{\bar{q}_1}{\psi(\tau' - \tau) + \tau - \phi\sigma^2} \end{aligned} \tag{A.49}$$

so that:

$$dq_1 = d\psi \left(\frac{(1 - \delta_0)(\tau' - \tau)}{\psi(\tau' - \tau) + \tau - \phi\sigma^2} - (\tau' - \tau) \frac{\bar{q}_1}{\psi(\tau' - \tau) + \tau - \phi\sigma^2} \right)$$

$$+\,dx\frac{\sigma^2}{\psi(\tau'-\tau)+\tau-\phi\sigma^2} \quad \text{(A.50)}$$

This gives us:

$$\frac{dr_\sigma}{dx_1} = -\frac{\sigma^2}{\psi(\tau'-\tau)+\tau-\phi\sigma^2} - \frac{d\psi}{dx_1}\left(\frac{(\tau'-\tau)(1-\delta_0-\bar{q})}{\psi(\tau'-\tau)+\tau-\phi\sigma^2}\right) \quad \text{(A.51)}$$

$$\frac{dr_\sigma}{dx_1} = -\frac{\sigma^2}{\psi(\tau'-\tau)+\tau-\phi\sigma^2}\left(1-\frac{d\psi}{dx_1}(\tau'-\tau)r_\sigma\right) \quad \text{(A.52)}$$

And we simply then use the fact that:

$$\frac{1}{\psi(\tau'-\tau)+\tau-\phi\sigma^2} = \frac{B_1-\phi(1-\delta_0)}{\psi(\tau'-\tau)+\tau-\phi\sigma^2}\frac{1}{B_1-\phi(1-\delta_0)} \quad \text{(A.53)}$$

which allows us to recover r_σ. thus leading to:

$$\frac{d\ln r_\sigma}{dx_1} = -\frac{1}{\sigma^2\big(B_1-\phi(1-\delta_0)\big)}\left(\sigma^2-(\tau'-\tau)r_\sigma\frac{d\psi}{dx_1}\right) \quad \text{(A.54)}$$

For the first-order approximation, use the second part of Proposition 3 to simply incorporate:

$$\frac{d\psi}{dx_1} = -\frac{\gamma\sigma\sqrt{\tau'-\tau}}{(2\psi(\tau'-\tau)+\tau-\phi\sigma^2)} \quad \text{(A.55)}$$

□

A.8 Proof of Corollary 1

This is simply coming form the supply curve of the sovereign:

$$\bar{b}_1 - B_1 - \phi\bar{q}_1 \implies \frac{d\bar{b}_1}{dx_1} = -\phi\frac{d\bar{q}_1}{dx_1} = \phi\frac{dr_\sigma}{dx_1} \quad \text{(A.56)}$$

A.9 Proof of Proposition 5

With asset purchases x_1, the new price in case of a negative signal (and agents only expect a single equilibrium at $t = 2$) is given by:

$$\frac{(1-\delta_0)(\psi\tau' + (1-\psi)\tau) - \psi\tau' s - \sigma^2 B_1 + \sigma^2 x_1}{\psi\tau' + (1-\psi)\tau - \phi\sigma^2} \tag{A.57}$$

So the new threshold is defined according to:

$$B_1 - \phi\frac{(1-\delta_0)(\psi_s\tau' + (1-\psi_s)\tau) - \psi\tau' s - \sigma^2 B_1 + \sigma^2 x_1}{\psi_s\tau' + (1-\psi_s)\tau - \phi\sigma^2} = b_1^* \tag{A.58}$$

$$(B_1 - b_1^*)(\psi_s\tau' + (1-\psi_s)\tau - \phi\sigma^2) = \phi\left((1-\delta_0)(\psi_s\tau' + (1-\psi_s)\tau) - \psi\tau' s - \sigma^2 B_1 + \sigma^2 x_1\right) \tag{A.59}$$

$$\begin{aligned}\psi_s\left((\tau'-\tau)(B_1 - b_1^*) - \phi(1-\delta_0)(\tau'-\tau) + \tau' s\right)& \\ = (\phi\sigma^2 - \tau)(B_1 - b_1^*) + \phi(1-\delta_0) - \phi\sigma^2(B_1 - x_1)&\end{aligned} \tag{A.60}$$

which simplifies to:

$$\psi_s(x_1) = \frac{\phi(1-\delta_0) - \tau(B_1 - b_1^*) - \phi\sigma^2(b_1^* - x_1)}{(\tau'-\tau)(B_1 - b_1^*) - \phi(1-\delta_0)(\tau'-\tau) + \tau' s} \tag{A.61}$$

and importantly:

$$\frac{\partial\psi_s(x_1)}{\partial x_1} > 0 \tag{A.62}$$

This means that it is sufficient to have expected utility being equal at the former threshold ($\psi_s(0)$ just denoted by ψ_s for simplicity) for the equilibrium to be unique after asset purchases. Going back to expected utilities:

$$\frac{\tau'-\tau}{2\sigma^2}\left[(1-\delta_0-\bar{q})^2 + \Psi_s^2 s^2 - 2\Psi_s s^2\right] + \frac{\tau'}{2\sigma^2}s^2 = \frac{\psi_s}{2\gamma^2} \tag{A.63}$$

Since Ψ_s is only a function of ψ_s, and not x_1, there is a single solution to this equation:

$$(1-\delta_0-\bar{q})^2 = \frac{1}{\tau'-\tau}\left(\frac{\sigma^2\psi_s}{\gamma^2} - \tau' s^2\right) - \Psi_s^2 s^2 + 2\Psi_s s^2 \tag{A.64}$$

$$\left(\frac{\sigma^2(B_1 - x_1^* - \phi(1-\delta_0))}{\psi_s\tau' + (1-\psi_s)\tau - \phi\sigma^2}\right)^2 = \frac{1}{\tau'-\tau}\left(\frac{\sigma^2\psi_s}{\gamma^2} - \tau' s^2\right) - \Psi_s^2 s^2 + 2\Psi_s s^2 \tag{A.65}$$

In particular, if asset purchases are big enough such that:

$$\left(\frac{\sigma^2(B_1 - x_1^* - \phi(1-\delta_0))}{\psi_s\tau' + (1-\psi_s)\tau - \phi\sigma^2}\right)^2 = \frac{1}{\tau'-\tau}\left(\frac{\sigma^2\psi_s^2}{\gamma^2}\right) + 2\Psi_s s^2 \tag{A.66}$$

then uniqueness is guaranteed since this is strictly larger than the unique solution to the previous equation equating expected utilities. This yields:

$$\frac{\sigma^2(B_1 - \bar{x}_1^* - \phi(1-\delta_0))}{\psi_s\tau' + (1-\psi_s)\tau - \phi\sigma^2} = \sqrt{\frac{1}{\tau'-\tau}\left(\frac{\sigma^2\psi_s^2}{\gamma^2}\right) + 2\frac{\psi_s\tau'}{\psi_s\tau' + (1-\psi_s)\tau - \phi\sigma^2}s^2} \tag{A.67}$$

To the first-order on s, an upper-bound on the size of asset purchases is simply given by assuring that:

$$\frac{\psi_s}{\gamma} = \frac{\sqrt{\tau'-\tau}(B_1 - \bar{x}_1 - \phi(1-\delta_0))}{\psi_s\tau' + (1-\psi_s)\tau - \phi\sigma^2} \tag{A.68}$$

which is:

$$\bar{x}_1 = (B_1 - \phi(1-\delta_0)) - \frac{\psi_s(\psi_s(\tau'-\tau) + \tau - \phi\sigma^2)}{\gamma\sqrt{\tau'-\tau}} \tag{A.69}$$

A.10 Proof of Proposition 6

Start with expected welfare:

$$\mathbb{E}V = \frac{(1+\beta)}{2}\ln\left(\frac{Y_2 - B_1 + \phi q_1(s)}{1+\beta}\right) + \frac{(1+\beta)}{2}\ln\left(\frac{Y_2 - B_1 + \phi q_1(-s)}{1+\beta}\right) \tag{A.70}$$

which yields the following derivative with respect to asset purchases:

$$\frac{d\mathbb{E}V}{dx_1} = \frac{\phi(1+\beta)}{2(Y_2 - B_1 + \phi q_1(s))}\frac{dq_1(s)}{dx_1} + \frac{\phi(1+\beta)}{2(Y_2 - B_1 + \phi q_1(-s))}\frac{dq_1(-s)}{dx_1} \tag{A.71}$$

Using again the formulation from the previous proofs, we can write it:

$$\frac{d\mathbb{E}V}{dx_1} = \frac{\phi(1+\beta)}{2}\frac{d\bar{q}_1}{dx_1}\left(\frac{1}{Y_2 - B_1 + \phi q_1(s)} + \frac{1}{Y_2 - B_1 + \phi q_1(-s)}\right)$$
$$+ \frac{\phi(1+\beta)}{2}\frac{d\Psi}{dx_1}s\left(\frac{1}{Y_2 - B_1 + \phi q_1(s)} - \frac{1}{Y_2 - B_1 + \phi q_1(-s)}\right) \quad \text{(A.72)}$$

The first derivative is equal to minus the derivative on the risk premium (since $r_\sigma = 1 - \delta_0 - \bar{q}_1$). I previously shown, this is ambiguous. The second term (second line) is itself unambiguously positive. Indeed, remember that we have:

$$\Psi = \frac{\psi\tau'}{\psi\tau' + (1-\psi)\tau - \phi\sigma^2} \quad \text{(A.73)}$$

so that Ψ is unambiguously increasing in ψ, and so is decreasing in x_1 (price sensitivity goes down after asset purchases, see Proposition 3). Then the term in parentheses is negative since $q_1(-s) < q_1(s)$.

The first-order approximation is then giving:

$$\frac{d\mathbb{E}V}{dx_1} = \frac{d\bar{q}_1}{dx_1}\frac{\phi(1+\beta)}{2}\left(\frac{1}{(Y_2 - B_1 + \phi q_1(s))} + \frac{1}{(Y_2 - B_1 + \phi q_1(-s))}\right) \quad \text{(A.74)}$$

and the symmetry of the expression in parenthesis implies that again to the first order:

$$\frac{d\mathbb{E}V}{dx_1} = \frac{d\bar{q}_1}{dx_1}\frac{\phi(1+\beta)}{Y_2 - B_1 + \phi\bar{q}_1} \quad \text{(A.75)}$$

Using our risk premium result from Proposition 4, this yields:

$$\frac{d\mathbb{E}V}{dx_1} = \frac{\phi(1+\beta)}{\bar{c}_2}\frac{r_\sigma}{\sigma^2\big(B_1 - \phi(1-\delta_0)\big)}\left(\sigma^2 + (\tau' - \tau)r_\sigma\frac{d\psi}{dx_1}\right) \quad \text{(A.76)}$$

A.11 Proof of Proposition 7

This is straightforward since Proposition 5 shows that big enough asset purchases shift the equilibrium to the left of the threshold ψ_s, and V_D is assumed to be low enough to ensure that welfare is higher without self-fulfilling debt crises.

A.12 Proof of Proposition 8

The proof is identical to the proof of Proposition 6, since the behavior of prices is identical.

B Suggestive Evidence on Information Production and Asset Purchases

One key prediction of the model presented in Section 2 is that asset purchases lower the equilibrium amount of information production (Proposition 3). In the formalization of the model this is saying that:

$$\frac{d\psi}{dx} < 0 \tag{B.1}$$

This Section seeks to verify whether the empirical evidence is consistent with this Proposition, specifically by looking at the motivating event of this paper: the ECB sovereign bond purchase program.

To this end, I gather analysts consensus forecasts on GDP, Inflation, and Industrial Production for Italy, from 2010 to 2023 (and ignore the forecasts made during the Covid period). I then construct the associated forecast errors for each forecast, and compute the variance of these forecast errors before and after the ECB sovereign bond purchase program (in 2015). Table 1 shows that the variance is more than twice a large after the ECB starts its purchasing program, suggesting that analysts are making less precise forecasts and thus not investing as much as before into information production.

Table 1: Variance of Analysts' Forecast Errors, Italy

2010 - 2015	2015-2023
0.327	**0.775**

C Alternative Setups

C.1 Market Orders

This section presents an alternative model where private investors can only condition on market prices when they pay the fixed cost associated with information acquisition. When this is the case, investors maximize instead:[25]

$$\mathbb{E}[U] = \mathbb{E}[b\chi - bq_1] - \frac{\sigma^2}{2}\mathbb{V}[b\chi] \tag{C.1}$$

This leads to the optimal portfolio choice:

$$b = \tau \frac{\mathbb{E}_1[\chi - q_1]}{\sigma^2} \tag{C.2}$$

Since investors that purchase information have the same demand function as in the benchmark model, the market clearing condition becomes, for any signal s:

$$B_1 - \phi q_1(s) = \psi\tau' \frac{1 - \delta_0 + s - q_1(s)}{\sigma^2} + (1-\psi)\tau \frac{1 - \delta_0 - \bar{q}_1}{\sigma^2} \tag{C.3}$$

This leads to the following expression:

$$q_1(s)\left(\psi\tau' - \phi\sigma^2\right) = \psi\tau'(1 - \delta_0 + s) + (1-\psi)(1 - \delta_0 - \bar{q}_1) - \sigma^2 B_1 \tag{C.4}$$

Taking the expectation on both sides to find the average price used by uninformed traders:

$$\bar{q}_1\left(\psi\tau' - \phi\sigma^2\right) = \psi\tau'(1 - \delta_0) + (1-\psi)\tau(1 - \delta_0 - \bar{q}_1) - \sigma^2 B_1 \tag{C.5}$$

which gives:

$$\bar{q}_1 = \frac{(1-\delta_0)(\psi\tau' + (1-\psi)\tau) - \sigma^2 B_1}{\psi\tau' + (1-\psi)\tau - \phi\sigma^2} \tag{C.6}$$

[25] This is implicitly assuming that investors have linear utility over period-1 consumption, and mean-variance preferences over period-2 consumption.

which has exactly the same expression as in the benchmark model. We can then plug that expression into the market clearing condition for any signal s:

$$q_1(s)\left(\psi\tau' - \phi\sigma^2\right) = \psi\tau'(1-\delta_0+s) + (1-\psi)\tau(1-\delta_0 - \frac{(1-\delta_0)(\psi\tau' + (1-\psi)\tau) - \sigma^2 B_1}{\psi\tau' + (1-\psi)\tau - \phi\sigma^2}) - \sigma^2 B_1 \quad \text{(C.7)}$$

which can be simplified to:

$$q_1(s) = \frac{(1-\delta_0)(\psi\tau' + (1-\psi)\tau) - \sigma^2 B_1}{\psi\tau' + (1-\psi)\tau - \phi\sigma^2} + s\frac{\psi\tau'}{\psi\tau' - \phi\sigma^2} \quad \text{(C.8)}$$

or for conciseness:

$$q_1(s) = \bar{q}_1 + s\frac{\psi\tau'}{\psi\tau' - \phi\sigma^2} \quad \text{(C.9)}$$

C.1.1 Noise Traders and Price Informativeness

This formulation makes clear that, in this alternative setup, prices are *less sensitive* to information when ψ is higher, an undesirable feature. For this reason, I also assume that there are noise traders with the following demand function:

$$\eta\frac{z - q_1}{\sigma^2} \quad \text{(C.10)}$$

with, on average, $\bar{z} = \bar{q}_1$. This way, the market clearing condition becomes:

$$B_1 - \phi q_1(s) = \psi\tau'\frac{1-\delta_0+s-q_1(s)}{\sigma^2} + (1-\psi)\tau\frac{1-\delta_0-\bar{q}_1}{\sigma^2} + \eta\frac{z-q_1}{\sigma^2} \quad \text{(C.11)}$$

Taking averages on both sides leads once again to the familiar:

$$\bar{q}_1 = \frac{(1-\delta_0)(\psi\tau' + (1-\psi)\tau) - \sigma^2 B_1}{\psi\tau' + (1-\psi)\tau - \phi\sigma^2} \quad \text{(C.12)}$$

Since q_1 is still obviously linear in s and z, we only need to find the coefficient in front of these random variables, thanks to:

$$-\phi\sigma^2\frac{dq_1}{ds} = -(\psi\tau' + \eta)\frac{dq_1}{ds} + \psi\tau' \quad \text{(C.13)}$$

$$-\phi\sigma^2 \frac{dq_1}{dz} = -(\psi\tau' + \eta)\frac{dq_1}{dz} + \eta \tag{C.14}$$

which yields the equilibrium price function:

$$q_1(s,z) = \bar{q}_1 + \frac{\psi\tau' s}{\psi\tau' + \eta - \phi\sigma^2} + \frac{\eta(z - \bar{q}_1)}{\psi\tau' + \eta - \phi\sigma^2} \tag{C.15}$$

And we indeed have, in this case, that prices are more responsive to information when ψ is greater, as long as we assume that $\eta > \phi\sigma^2$. We now focus on the realization $z = \bar{q}_1$ since we are interested in the information channel and not on noise traders. Rewrite this price function, for simplicity, as:

$$q_1(s) = \bar{q}_1 + \Psi s \tag{C.16}$$

C.1.2 Roll-Over Threshold

Given this equilibrium price, the threshold is found when (as in the benchmark model):

$$B_1 - \phi\frac{(1-\delta_0)(\psi_s\tau' + (1-\psi_s)\tau) - \sigma^2 B_1}{\psi_s\tau' + (1-\psi_s)\tau - \phi\sigma^2} + \phi\frac{\psi_s\tau' s}{\psi_s\tau' + \eta - \phi\sigma^2} = b^* \tag{C.17}$$

We also assume a condition similar to Assumption 1 in order to get: $\frac{dq_1(-s)}{d\psi} < 0$, such that there is at most one threshold ψ_s.

C.1.3 Information Choice

Since Uninformed investors know that they will only be able to submit a demand for bonds that does not condition on equilibrium prices, their expected utility is given by:

$$\mathbb{E}[U|\psi < \psi_s] = \tau\frac{(1-\delta_0 - \bar{q}_1)^2}{2\sigma^2} \tag{C.18}$$

This expression shows why this alternative setup is attractive: uninformed agents know that they will not be able to condition on average prices, so this expression does not contain expectations over prices in different signal realizations, as we had in the main framework. For informed traders, the expression is similar as in the main framework since their demand function is unchanged:

$$\mathbb{E}[U_\gamma|\psi < \psi_s] = \frac{\tau'}{4\sigma^2}\left[(1-\delta_0+s-q_1(+s))^2 + (1-\delta_0-s-q_1(-s))^2\right] - \frac{i^2}{\gamma^2} \quad \text{(C.19)}$$

Rewrite this as:

$$\mathbb{E}[U_\gamma|\psi < \psi_s] = \frac{\tau'}{4\sigma^2}\left[(1-\delta_0-\bar{q}_1+(1-\Psi)s)^2 + (1-\delta_0-\bar{q}_1-(1-\Psi)s)^2\right] - \frac{i^2}{\gamma^2} \quad \text{(C.20)}$$

Which can be simplified to:

$$\mathbb{E}[U_\gamma|\psi < \psi_s] = \frac{\tau'}{2\sigma^2}\left[(1-\delta_0-\bar{q}_1)^2 + (1-\Psi)^2 s^2\right] - \frac{i^2}{\gamma^2} \quad \text{(C.21)}$$

If the equilibrium ψ is indeed below the threshold ψ_s, it thus verifies:

$$(\tau'-\tau)(1-\delta_0-\bar{q}_1)^2 + \tau' s^2(1-\Psi)^2 = \frac{\sigma^2\psi^2}{\gamma^2} \quad \text{(C.22)}$$

Since both components of the left-hand side of this equality are strictly decreasing (since $\Psi < 1$ and $\bar{q}_1 < 1-\delta_0$ for all ψ) in ψ, while the right-hand side is strictly increasing in ψ, there is at most one solution to this equation. The insights of the main model are then unchanged.

C.2 Learning from Bond Prices

The main framework presented in the core of the paper assumed that traders that do not pay the fixed cost are unable to learn the signal received by other traders. This extensions shows that this assumption is not driving the main result of the paper. To ensure a tractable structure, assume the following: each trader i receives a signal s_i. Each idiosyncratic signal is a noisy signal of the aggregate signal s as in the main framework, $s_i = s + \epsilon_i$, and we denote by g_l the associated Kalman gain coefficient, relative to the case when traders receive s.[26] As such, an agent

[26] I am slightly abusing notations here to keep the framework as close as possible to the main one presented in the core of the paper. The signal s is originally defined as by how much it moves the posterior relative to the prior (which is not exactly the same thing as what is the information received: going from one to the other requires the Kalman gain. When traders only receive a noisy

whose only information comes from the private signal has expectations: $\mathbb{E}_i[\chi] = (1-\delta_0) + gs_i$. We then assume that, in order to be able to learn from prices, investor i must pay a fixed cost i^2/γ^2.

Because prices are fully revealing, a trader that learns from prices is always able to fully recover the aggregate signal s, and hence trade without the idiosyncratic noise. We can thus define the precision levels τ_l and τ_l' as the ones used by traders when they trade on $s+\epsilon_i$ and when they trade on s. The market clearing condition is thus:

$$B_1 - \phi q_1(s) = \psi\tau_l' \frac{1-\delta_0+s-q_1(s)}{\sigma^2} + \int_\psi^1 \tau_l \frac{1-\delta_0+g_l(s+\epsilon_i)-q_1(s)}{\sigma^2} di \tag{C.23}$$

And given the assumption that ϵ_i is idiosyncratic noise, they cancel on the aggregate such that:

$$B_1 - \phi q_1(s) = \psi\tau_l' \frac{1-\delta_0+s-q_1(s)}{\sigma^2} + (1-\psi)\tau_l \frac{1-\delta_0+g_l s-q_1(s)}{\sigma^2} \tag{C.24}$$

and the equilibrium price is given by :

$$q_1(s_1;\psi) = \frac{(1-\delta_0)(\psi\tau_l' + (1-\psi)\tau_l) + (\psi\tau' + (1-\psi)g_l\tau_l')s - \sigma^2 B_1}{\psi\tau_l' + (1-\psi)\tau_l - \phi\sigma^2} \tag{C.25}$$

The equilibrium is thus similar: the only change is the expression for the price sensitivity to information. It is an increasing function of ψ when:

$$\frac{d}{d\psi}\left(\frac{\psi\tau' + (1-\psi)g_l\tau_l'}{\psi\tau_l' + (1-\psi)\tau_l - \phi\sigma^2}\right) = \frac{\tau_l'\tau_l(1-g_l) - (\tau_l' - \tau_l g_l)\phi\sigma^2}{(\psi\tau_l' + (1-\psi)\tau_l - \phi\sigma^2)^2} > 0 \tag{C.26}$$

where we had that, by definition, $g_l < 1$ and $\tau_l' > \tau_l$. When $g_l = 0$ (as in the benchmark framework) then the condition was equivalent to:

$$\tau > \phi\sigma^2 \tag{C.27}$$

---

signal, they update in the direction of the signal but less than in the case where they received the signal without idiosyncratic noise. To keep notation consistent, I thus define the Kalman gain g as the scalar that allows me to write the posterior as a function of s_i. g_l is then intuitively less than 1.

Whereas now it can be written:

$$\tau_l \left(\frac{\tau_l' - \tau_l' g_l}{\tau_l' - \tau_l g_l} \right) > \phi \sigma^2 \tag{C.28}$$

which is more restrictive when $g_l > 0$. We assume that this condition holds.

D Additional Results

D.1 Linear Supply Assumption

The main framework assumed a linear supply of bonds in order to achieve tractable insights:

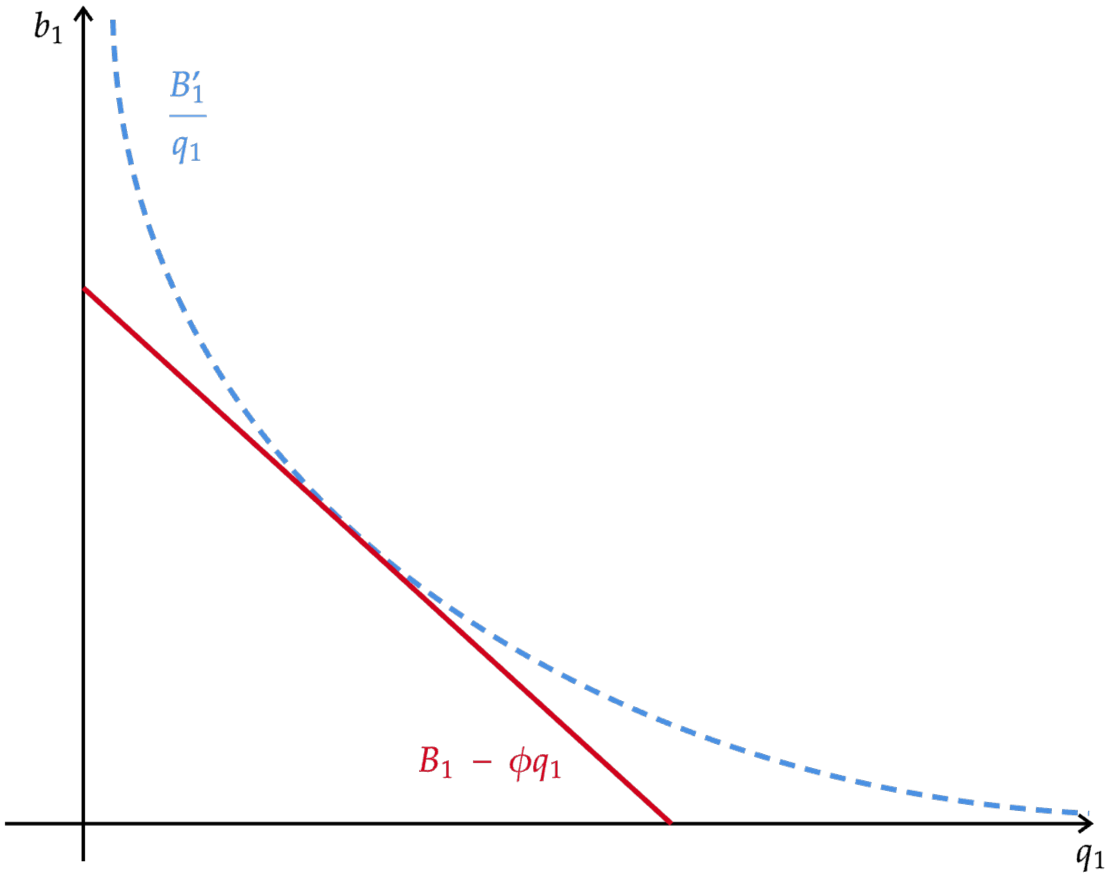
$$b_1 = B_1 - \phi q_1. \tag{D.1}$$

Section 5 also shows that these insights are robust to assuming more conventional supply elasticities. Another way to understand why this assumption is innocuous is depicted in Figure 13. This figure shows that, by choosing carefully the parameters B_1 and ϕ, one is left with a supply curve that always lays below the case with a fixed expenditure to finance, B_1'/q_1. In that case, for any equilibrium price q_1, the country issues even more bonds under the fixed expenditure case. Intuitively, this means that the linear supply assumption is a conservative one when it comes to the analysis of roll-over crises in this setup.

D.2 Case with ψ_V

Figure 14 shows an example where psi_V exists. In this case, the welfare of the sovereign is greater with roll-over crises than without, but only when information acquisition is far greater than at the threshold. Intuitively, in this case asset purchases are valuable when the equilibrium level of information production is contained in $[\psi_s, \psi_V]$.

Figure 13: Different Supply Assumptions. The red line corresponds to a linear supply of sovereign bonds as in the main framework of Section 2. The dashed blue line is the case where the sovereign has no other choice than to raise a quantity B_1'.



E Uncertain Scale of Asset Purchases

Should central bankers seek to communicate very clearly the scale of future purchases, or is uncertainty about their actions a desirable feature? We can answer that question in the framework of Appendix C.1, for cleaner expressions. Recall that, with market orders, uninformed agents hold:

$$b_U = \tau \frac{1 - \delta_0 - \bar{q}_1}{\sigma^2} \tag{E.1}$$

while information choice was dictated by the following expected utility term:

$$\mathbb{E}[U|\psi < \psi_s] = \tau \frac{(1-\delta_0-\bar{q}_1)^2}{2\sigma^2} \tag{E.2}$$

This expression for uninformed traders is completely unchanged by the presence of uncertainty on asset purchases. This is because expected utility is coming from:

$$\mathbb{E}[U|\psi < \psi_s] = b_U \mathbb{E}[\chi - q_1] - b_U^2 \mathbb{V}[\chi] \tag{E.3}$$

And for a given ψ, prices are linear in x_1 so uncertainty is not relevant here. For informed agents, however, the expected utility calculation is changed by the presence of uncertainty over x_1, since their positions will be different for different values of x_1. We can write their expected utility as:

$$\mathbb{E}[U_\gamma|\psi < \psi_s] = \frac{\tau'}{2\sigma^2} \mathbb{E}\left[(1-\delta_0+s_1-q_1(s_1,x_1))\right] \tag{E.4}$$

Assume now that x_1 can take two values: $x+\tilde{x}$ and $x-\tilde{x}$. To the first order in s, and second order in $\tilde{x}$, we have that:

$$\mathbb{E}[U_\gamma|\psi < \psi_s] = \frac{\tau'}{2\sigma^2}\left((1-\delta_0-\bar{q}_1)^2 + \left(\frac{\sigma^2}{\psi-\phi\sigma^2}\right)\tilde{x}^2\right) \tag{E.5}$$

which is increasing in $\tilde{x}$. As such, for a fixed ψ, the introduction of uncertainty over x_1 increases the expected utility of informed traders while keeping it unchanged for uninformed traders. As a result, this pushes for more information acquisition, the opposite of the effect sought by the central bank. In conclusion, reducing the uncertainty over the scale of future asset purchases is desirable for central bankers that seek to reduce information acquisition.

Figure 14: Sovereign's value function at $t = 2$ when ψ_V exists. The blue lines represent how the value function changes with the amount of information acquisition realized at $t = 1$, for a positive and a negative signal. The red line is the expectation over the signal realization, and over the realization of the sunspot λ. The blue line is the expected value in the case where the sunspot coordinates an equilibrium with repayment. The blue dotted line is the value upon defaulting. The discontinuity arises for a negative signal because of a rollover crisis possibility when $\psi > \psi_s$. For $\psi > \psi_V$, the expected welfare of the sovereign is higher with roll-over crises possibility than without because a large amount of informed investors reduces the equilibrium risk premium enough to compensate for the risk of roll-over crises.